



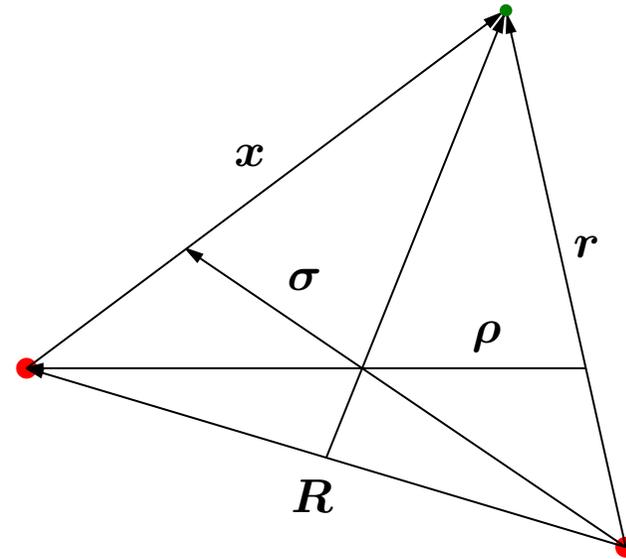
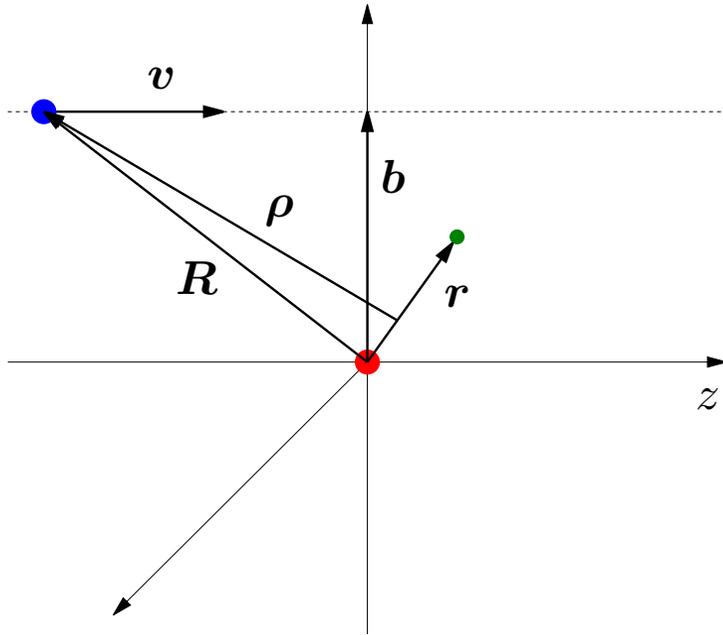
Curtin University

Work done over the Neutral Beams CRP using the CCC approach

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2-centre CCC approach



2 sets of Jacobi coordinates

$H\Psi_i^+ = E\Psi_i^+$ this is the total scattering wave function (not electronic)

$$\Psi_i^+ = \sum_{\alpha=1}^N F_{\alpha}(t, \mathbf{b}) e^{i\mathbf{q}_{\alpha} \cdot \boldsymbol{\rho}} \psi_{\alpha}(\mathbf{r}) + \sum_{\beta=1}^M G_{\beta}(t, \mathbf{b}) e^{i\mathbf{q}_{\beta} \cdot \boldsymbol{\sigma}} \psi_{\beta}(\mathbf{x})$$

A new ansatz that doesn't require so-called electron translation factors (ETF)

2-centre CCC approach

$$\left\{ \begin{array}{l} i\dot{F}_{\alpha'} + i \sum_{\beta=1}^M \dot{G}_{\beta} \tilde{K}_{\alpha'\beta} = \sum_{\alpha=1}^N F_{\alpha} D_{\alpha'\alpha} + \sum_{\beta=1}^M G_{\beta} \tilde{Q}_{\alpha'\beta} \\ i \sum_{\alpha=1}^N \dot{F}_{\alpha} K_{\beta'\alpha} + i\dot{G}_{\beta'} = \sum_{\alpha=1}^N F_{\alpha} Q_{\beta'\alpha} + \sum_{\beta=1}^M G_{\beta} \tilde{D}_{\beta'\beta} \end{array} \right.$$

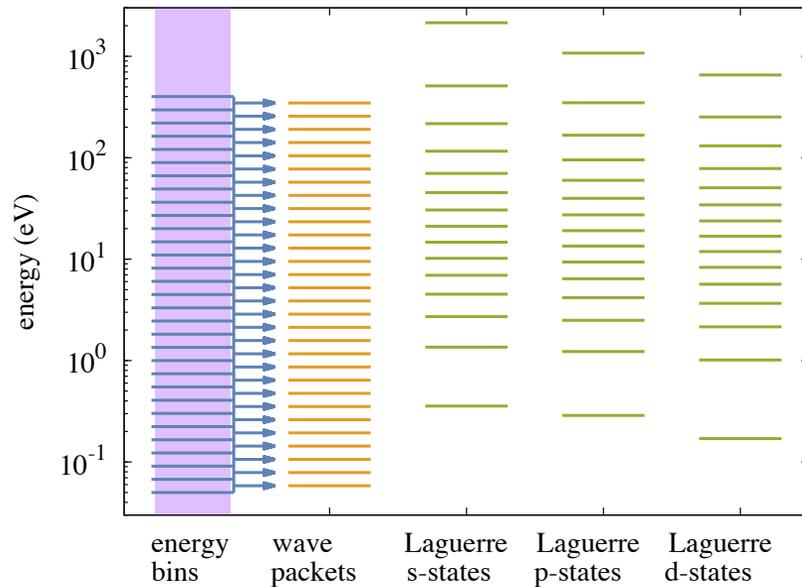
Probability amplitudes

$$A_{\alpha}^{\text{DS}}(\mathbf{b}) = F_{\alpha}(+\infty, \mathbf{b}) - \delta_{\alpha i}$$

$$A_{\beta}^{\text{EC}}(\mathbf{b}) = G_{\beta}(+\infty, \mathbf{b})$$

Direct scattering (DS): elastic scattering or target excitation including continuum
Electron capture (EC): this could be capture to bound state or continuum (ECC)

Wave-packet continuum discretisation



$$\phi_{nl}(r) = \frac{1}{\sqrt{w_n}} \int_{k_{n-1}}^{k_n} dk U_l(k, r)$$

Coulomb wave

For n^{th} bin the wave packet is at

$$\epsilon_n = (k_n^2 + k_n k_{n-1} + k_{n-1}^2) / 6$$

Advantages of WPs:

- Flexible density and distribution
- WPs aligned for different ℓ
- Overlap with Coulomb wave is simple

$$\langle \psi_{\kappa}^- | \psi_f^{\text{WP}} \rangle = \sqrt{\frac{2}{\pi}} (-i)^l e^{i\sigma_l} b_{nl}(\kappa) Y_{lm}(\hat{\kappa})$$

$$b_{nl}(\kappa) = \int_0^{\infty} dr U_l(\kappa, r) \phi_{nl}(r) = \frac{1}{\sqrt{w_n}}$$

p – He collisions (4-body problem)

$$\Psi = \sum_{\alpha=1}^N F_{\alpha}(t, \mathbf{b}) \psi_{\alpha}^{\text{He}}(\mathbf{r}_1, \mathbf{r}_2) e^{i\mathbf{q}_{\alpha} \cdot \boldsymbol{\rho}}$$
$$+ \frac{1}{\sqrt{2}} \sum_{\beta=1}^M G_{\beta}(t, \mathbf{b}) \left[\psi_{\beta}^{\text{H}}(\mathbf{x}_1) \psi_{1s}^{\text{He}^+}(\mathbf{r}_2) e^{i\mathbf{q}_{1\beta} \cdot \boldsymbol{\sigma}_1} + \psi_{\beta}^{\text{H}}(\mathbf{x}_2) \psi_{1s}^{\text{He}^+}(\mathbf{r}_1) e^{i\mathbf{q}_{2\beta} \cdot \boldsymbol{\sigma}_2} \right]$$

Target structure (3-body problem) in the CI-FC approximation

$$\psi_{\alpha}^{\text{He}}(\mathbf{r}_1, \mathbf{r}_2) = \psi_{\alpha}(\mathbf{r}_1) \psi_{1s}(\mathbf{r}_2) + \psi_{\alpha}(\mathbf{r}_2) \psi_{1s}(\mathbf{r}_1)$$

$$H_T \psi_{\alpha}^{\text{He}}(\mathbf{r}_1, \mathbf{r}_2) = E_{\alpha} \psi_{\alpha}^{\text{He}}(\mathbf{r}_1, \mathbf{r}_2)$$

This gives

- Negative-energy discrete states and
- Positive-energy continuum solution (non-normalisable wave function)

The continuum solution is used to generate WP

Integrated cross sections

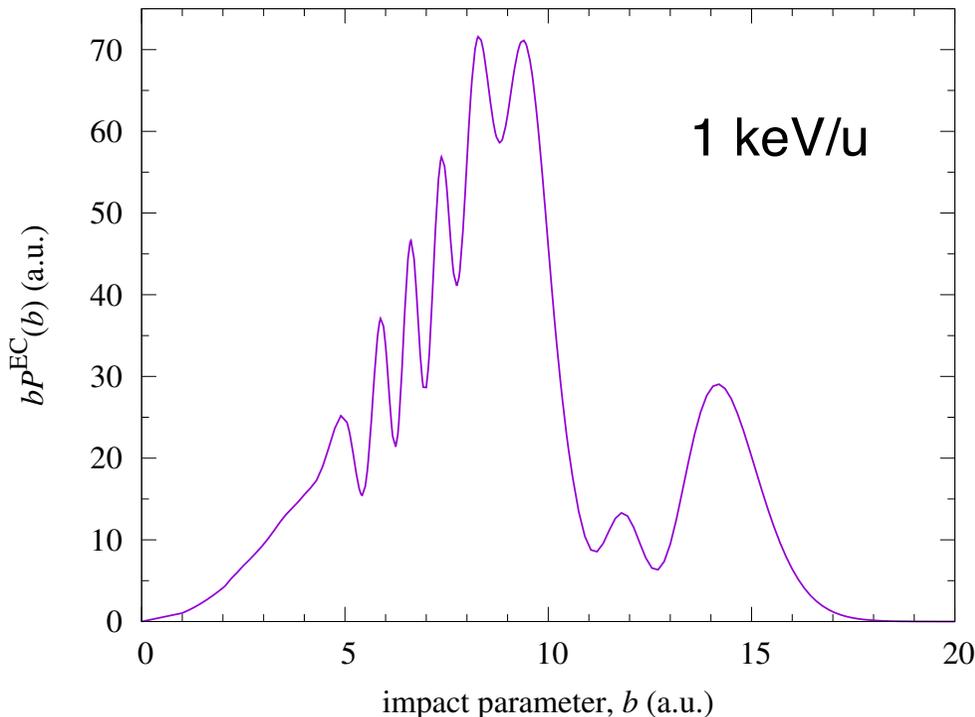
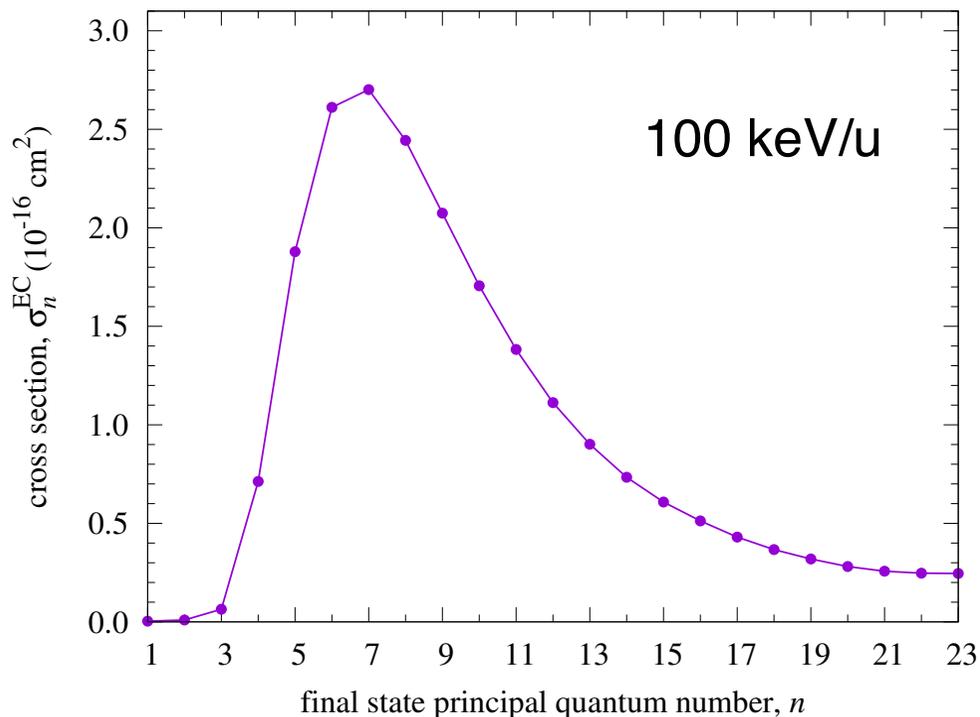
$$\sigma_f = 2\pi \int_0^\infty db \, b |A_f(b)|^2, \quad f = \alpha, \beta$$

$$\sigma_{\text{TECS}} = \sum_{\beta \in [\epsilon_\beta < 0]}^M \sigma_\beta^{\text{EC}}$$

$$\sigma_{\text{TICS}} = \sum_{\alpha \in [\epsilon_\alpha > 0]}^N \sigma_\alpha^{\text{DS}} + \sum_{\beta \in [\epsilon_\beta > 0]}^M \sigma_\beta^{\text{EC}}$$

- We increase N and M until the results converge not only for the total electron capture cross section (TECS) and total ionisation cross section (TICS) but also for most important state-selective cross sections
- Use GPU-based supercomputers
- Calculated: p, He²⁺, Be⁴⁺, C⁶⁺, Ne¹⁰⁺ on H
p on He, Li, Na, K and H₂

Electron capture and ionisation in Ne^{10+} - $\text{H}(1s)$



Energy range considered: 1 keV/u to 2 MeV/u.

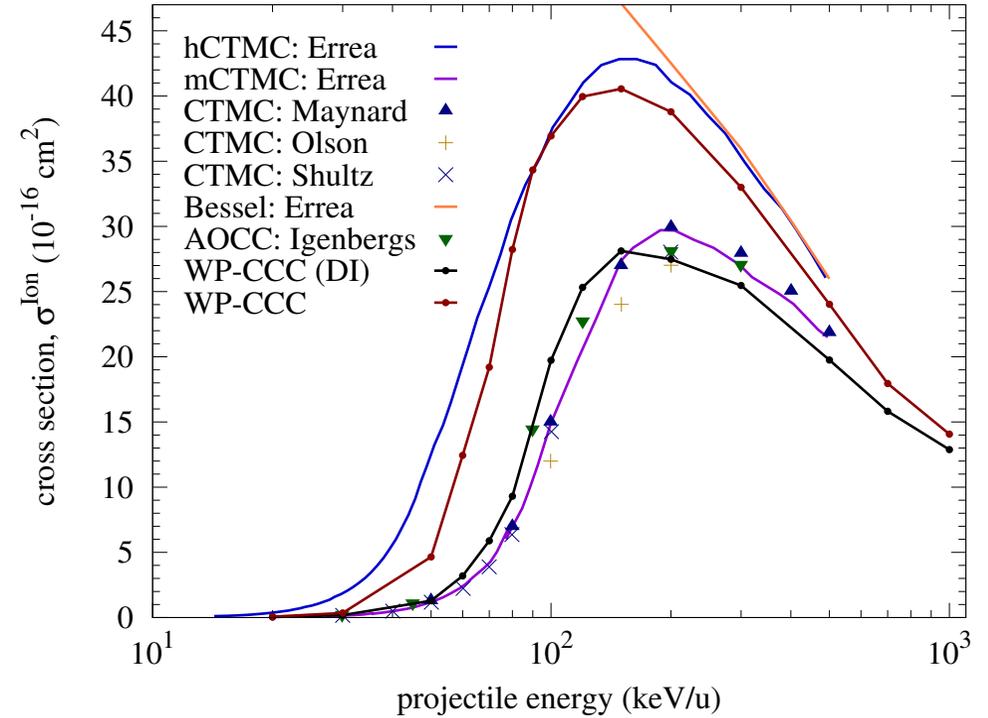
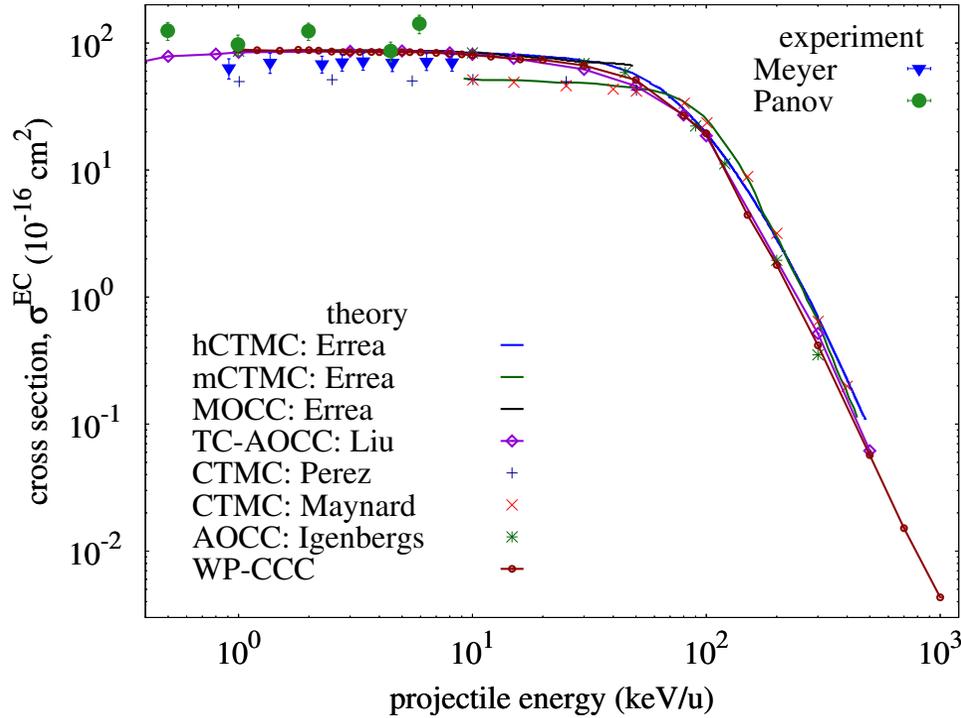
The results have converged within a few % over the entire energy range.

Usually, for low Z ions $n_{\text{max}} \approx Z$, but for HCI we find $n_{\text{max}} > Z$

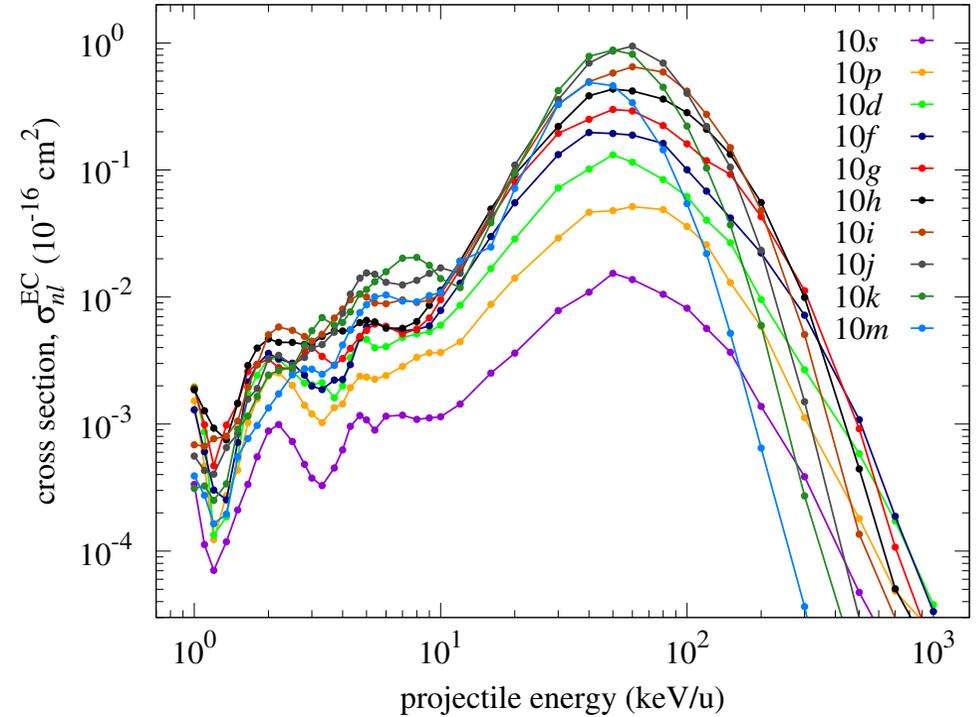
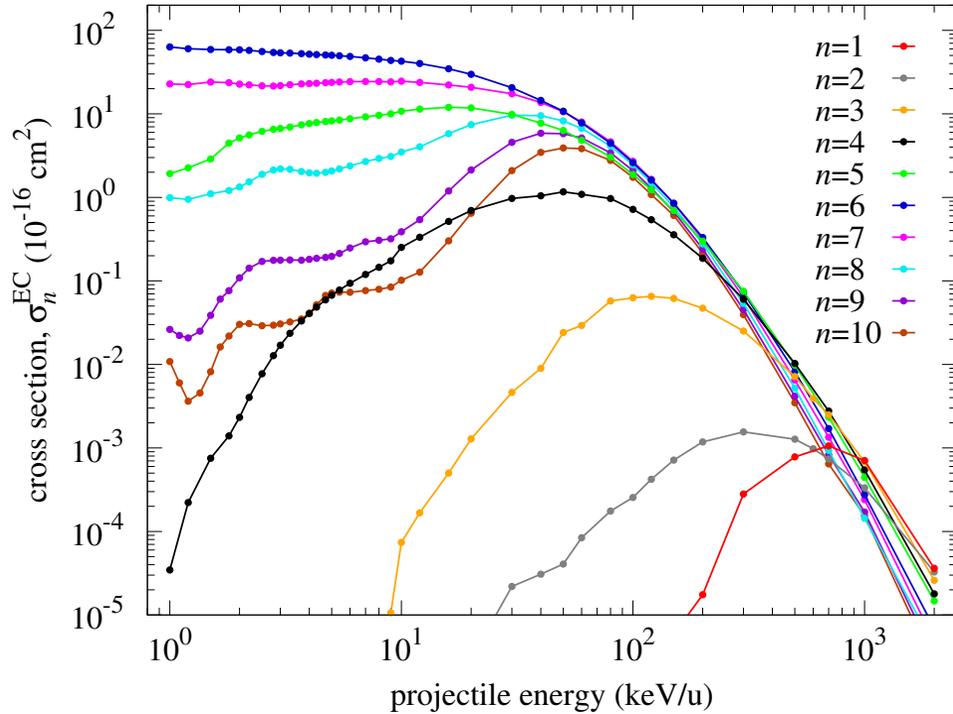
At 100 keV/u we had to go up to $n = 23$ to achieve a few % convergence.

The number of WP states was $N_c = 15$ and $\ell_{\text{max}} = 9$.

Total capture and ionisation in Ne^{10+} - H(1s)



State-selective electron capture in Ne^{10+} - $\text{H}(1s)$

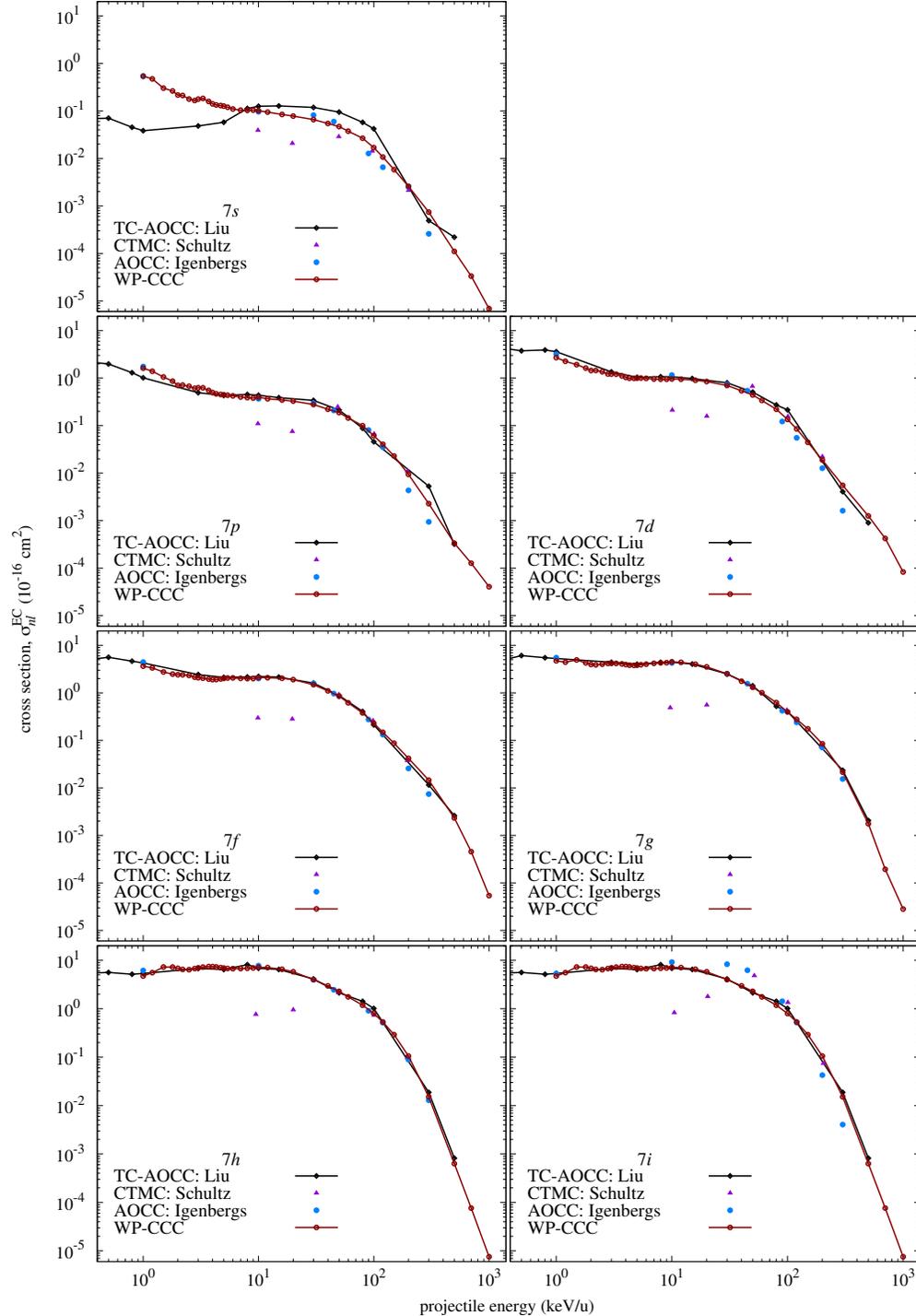
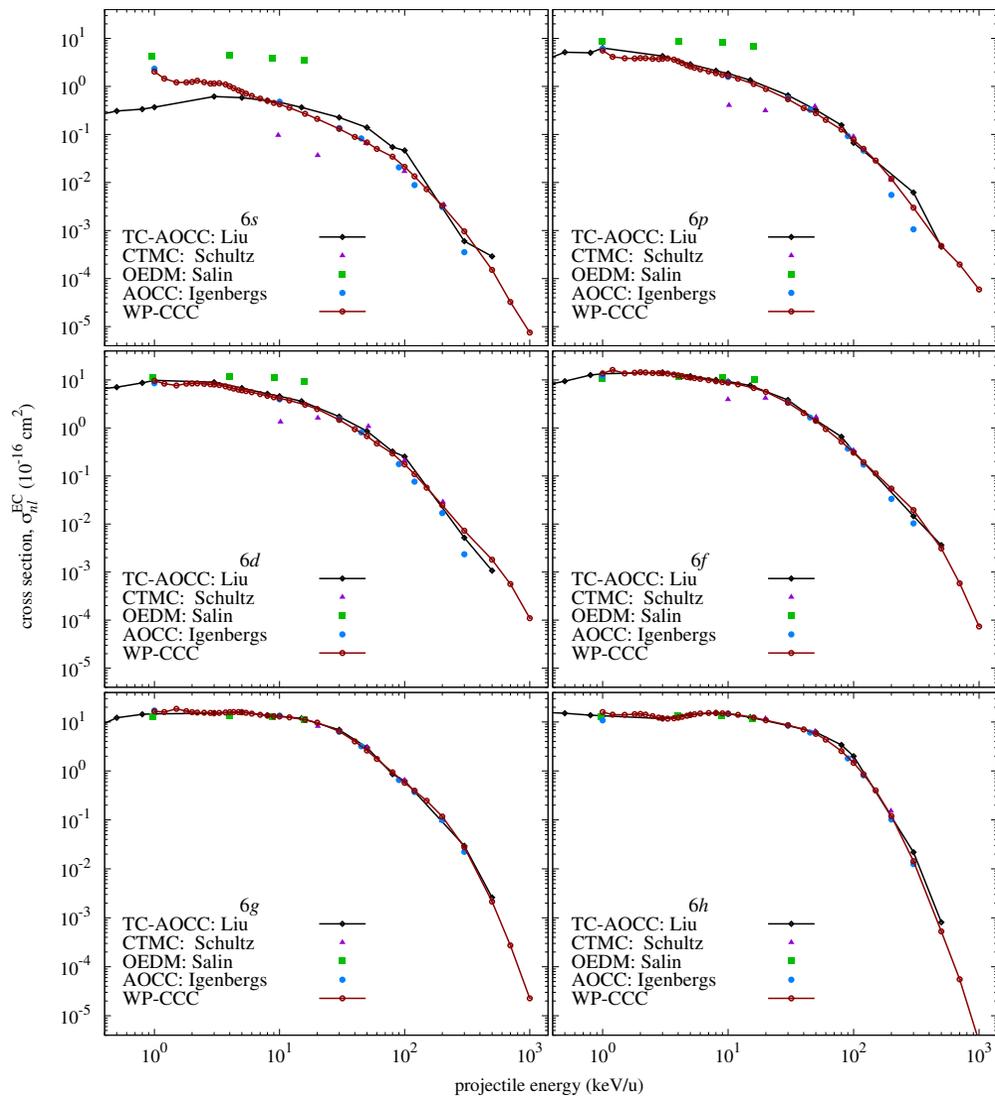


Oscillations in the 10ℓ partial cross sections. Only in non-resonant transitions. See Schultz et al., PRL 78, 2720 (1997).

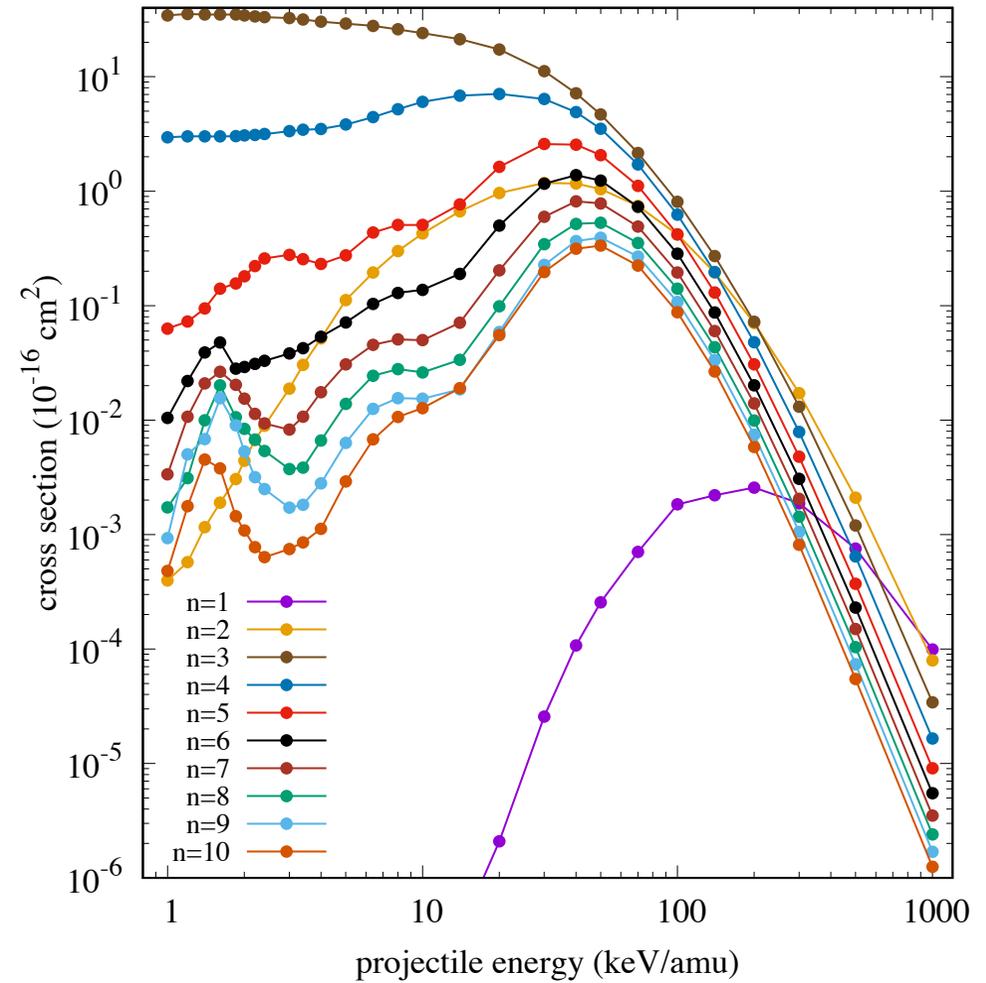
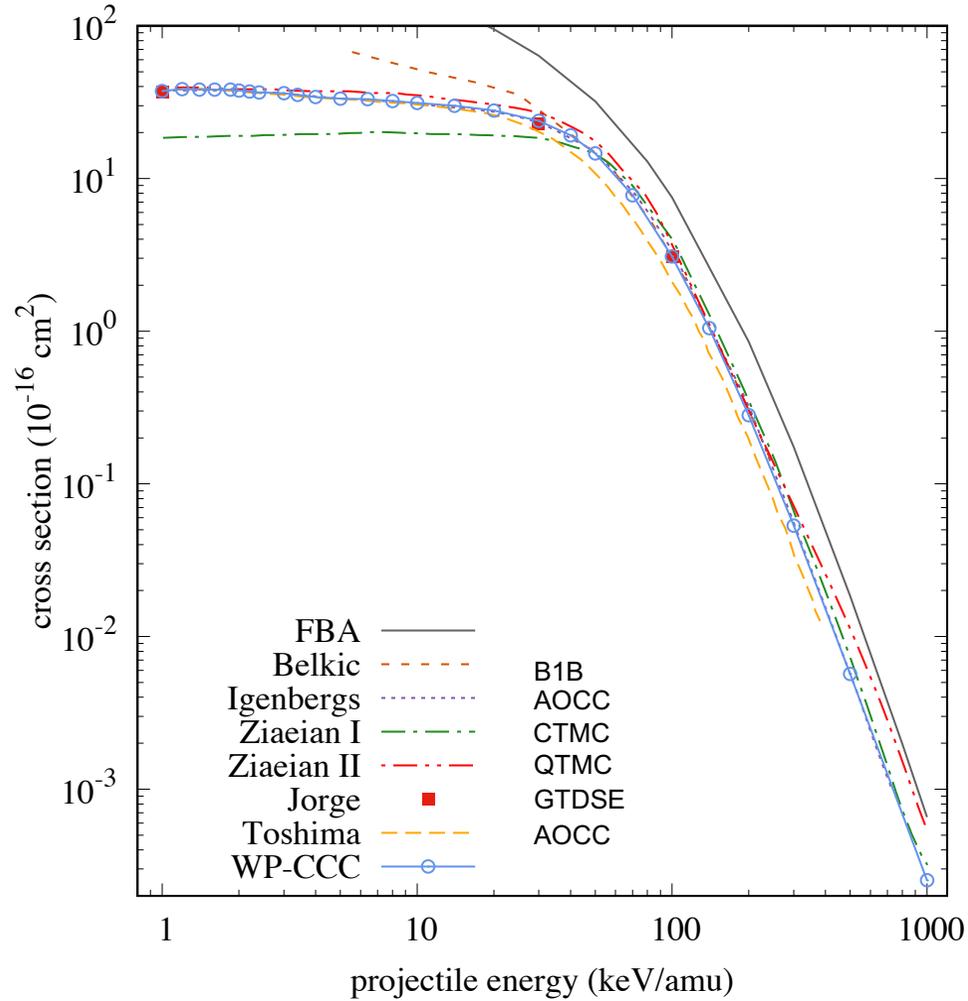
Kotian et al. 2022 J. Phys. B 55, 115201

Ne¹⁰⁺ - H(1s) → Ne⁹⁺(nl) - H⁺

Our results deviate from TC-AOCC by Liu *et al* for 6l, 7l, 8l below 5 keV



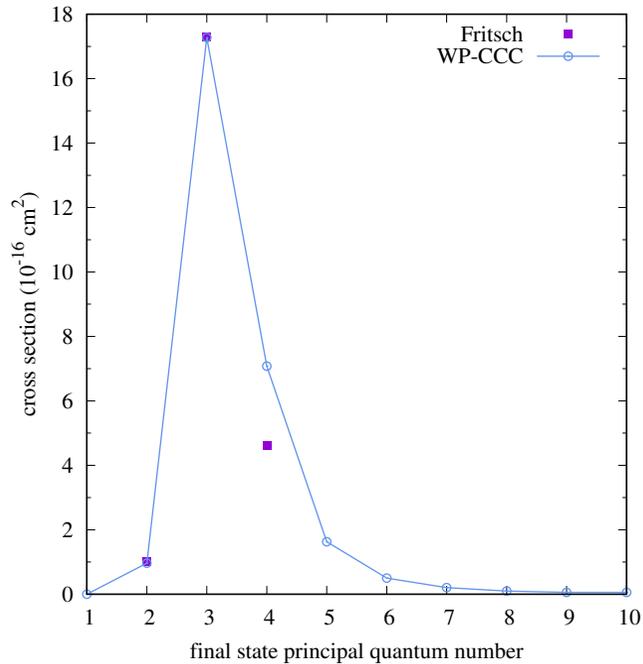
Total and state-selective capture in $\text{Be}^{4+} - \text{H}(1s)$



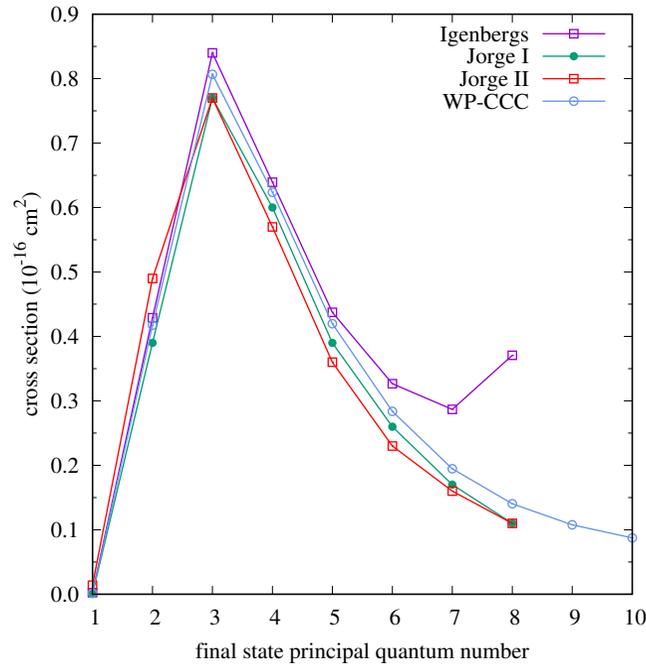
$\text{Be}^{4+} - \text{H}(1s)$: Antonio et al 2021 J. Phys. B 54, 175201

State-selective capture in $\text{Be}^{4+} - \text{H}(1s)$

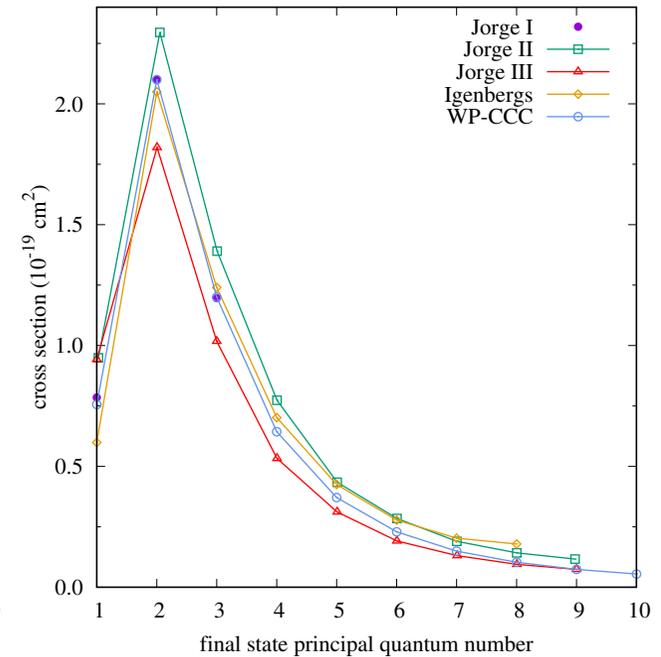
20 keV



100 keV

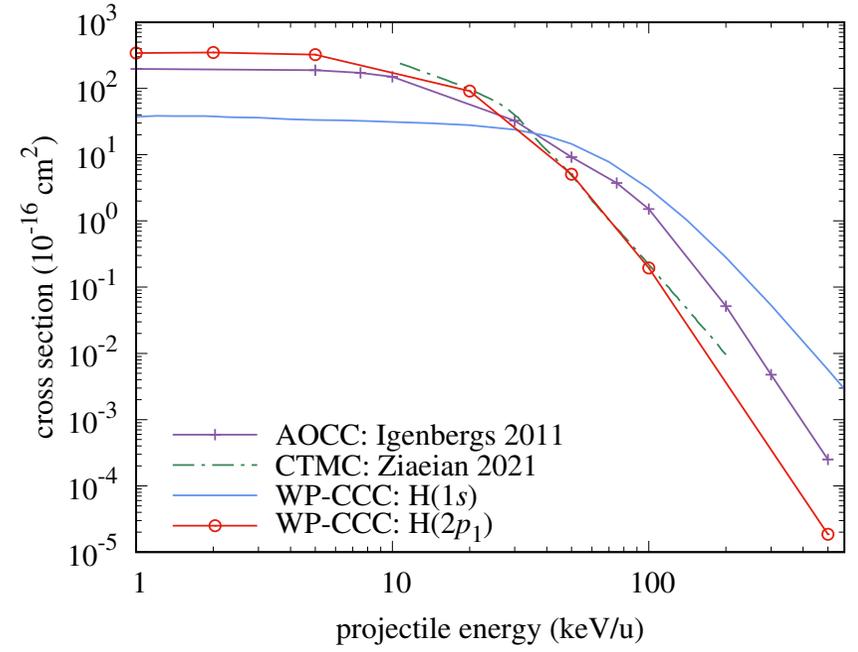
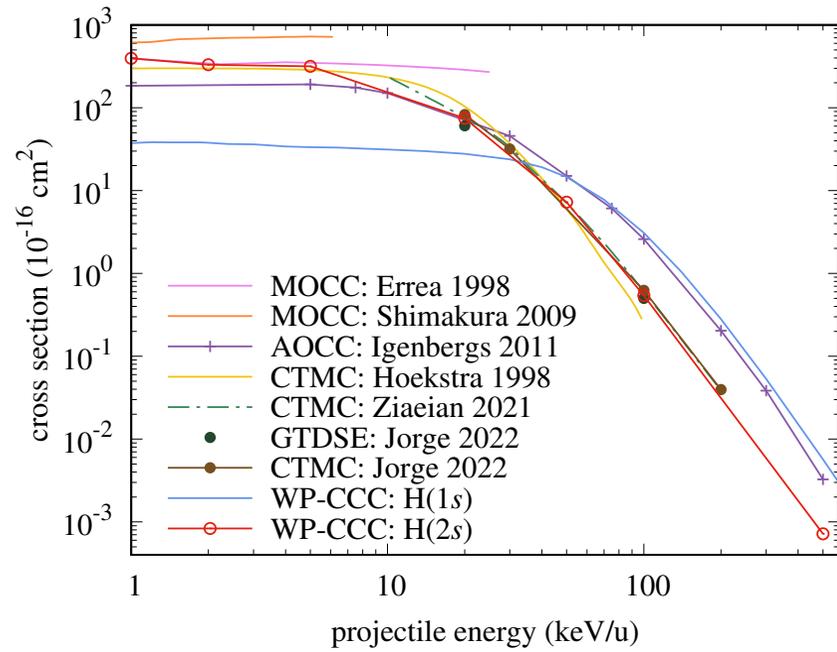
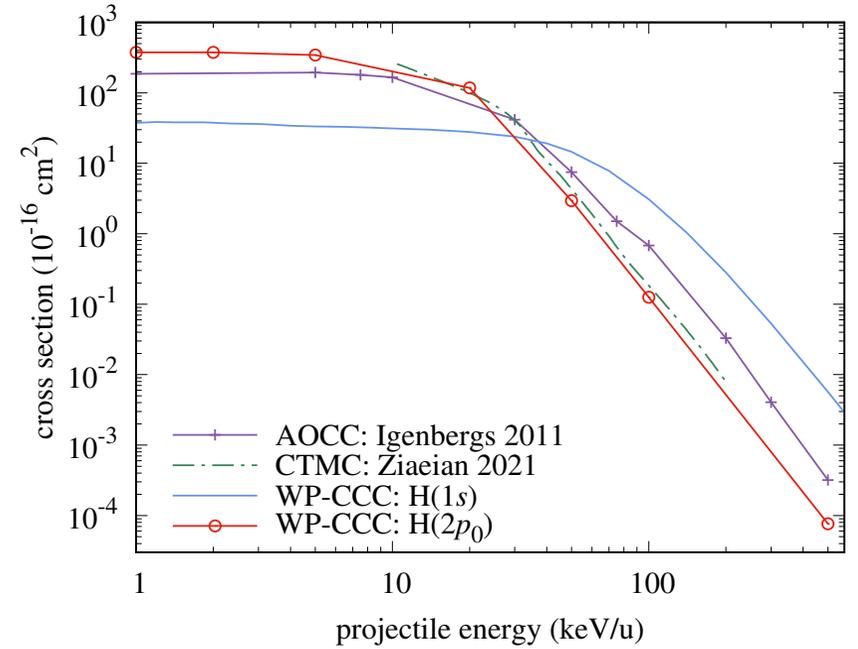
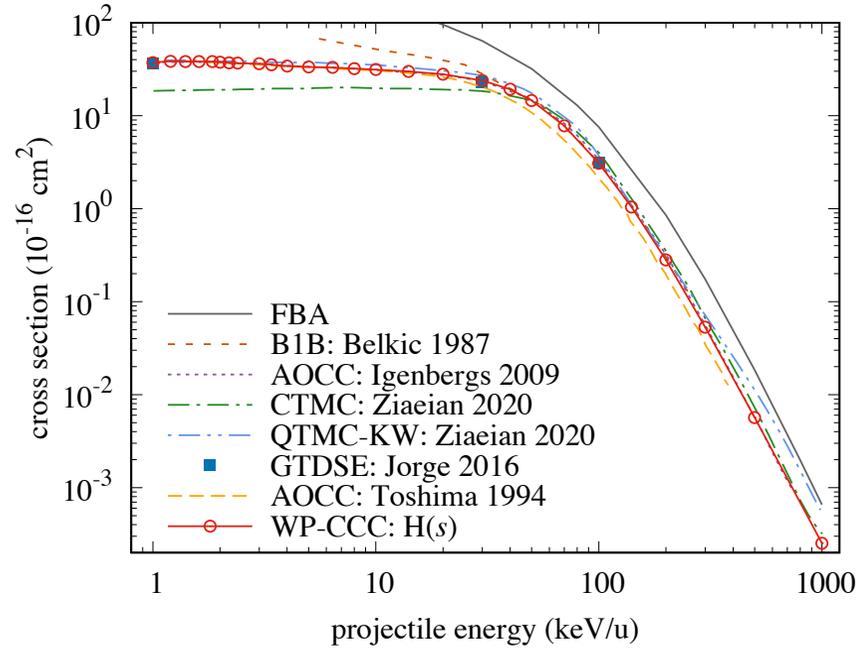


500 keV

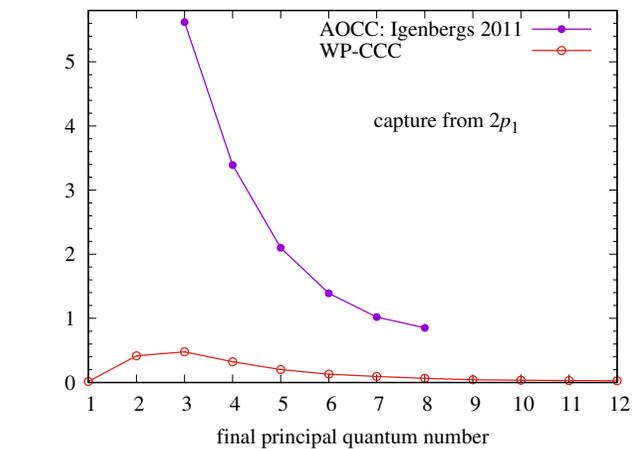
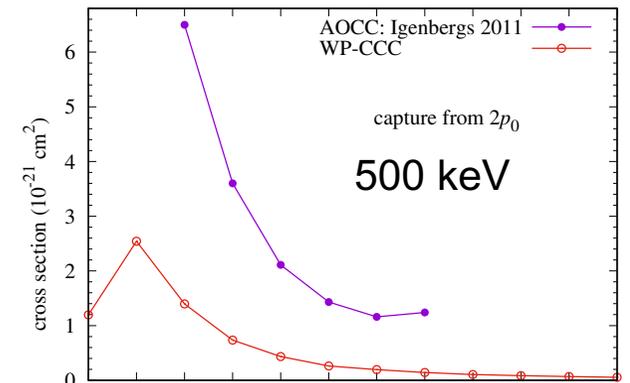
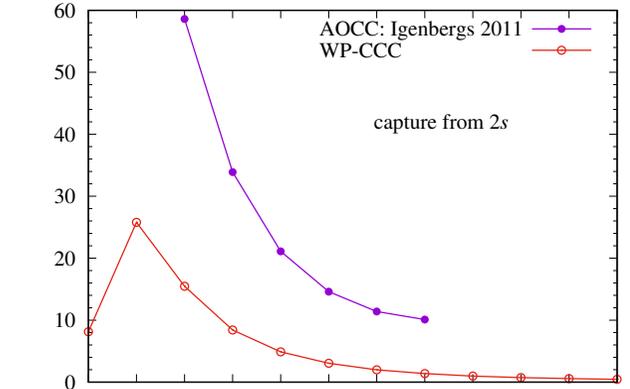
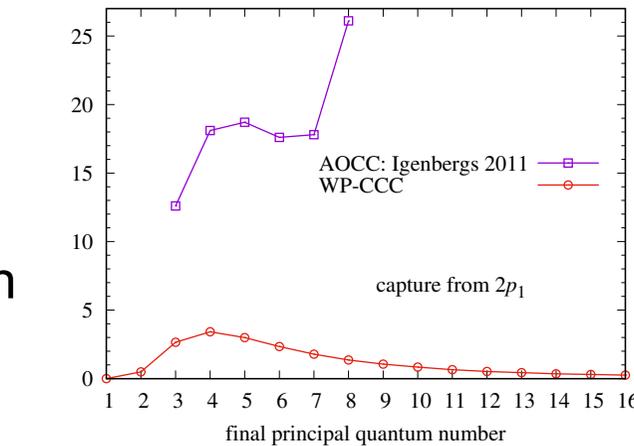
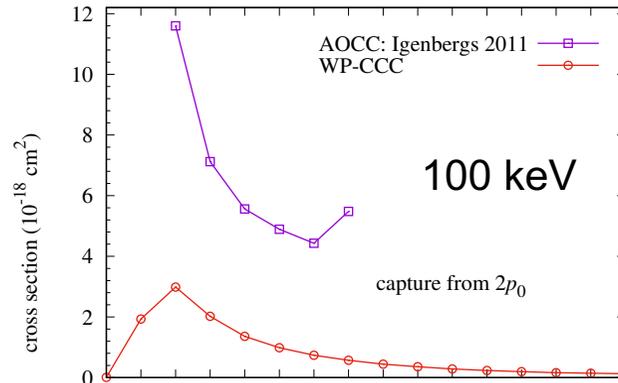
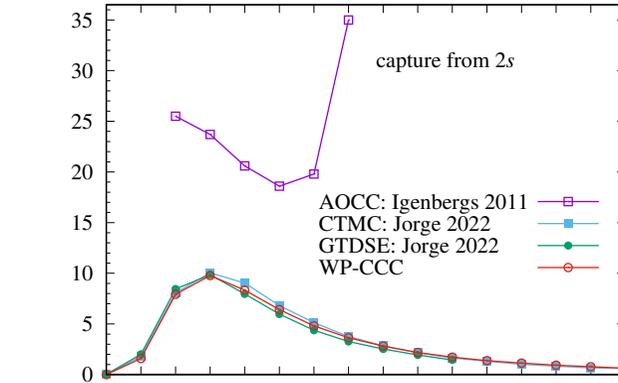
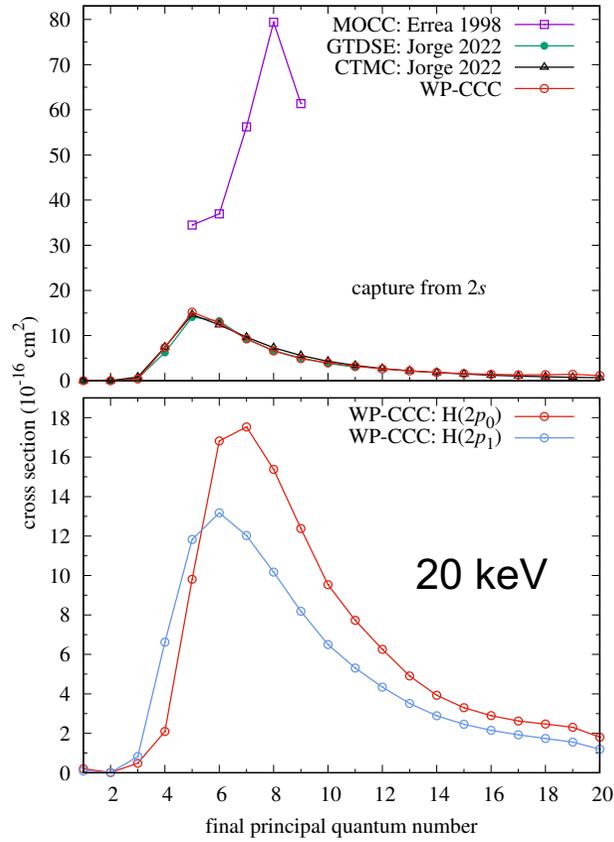


$\text{Be}^{4+} - \text{H}(1s)$: Antonio et al 2021 J. Phys. B 54, 175201

Total capture in Be^{4+} - H(1s) and Be^{4+} - H(2lm)



Total and state-selective capture in $\text{Be}^{4+} - \text{H}(2\text{Im})$



Antonio et al,
submitted for publication

Summary of completed work

- Developed 2-centre CCC approach to ion-atom collisions
- Calculated integrated total and state-selective cross sections for
 - p, He²⁺, Be⁴⁺, C⁶⁺, Ne¹⁰⁺ - H(1s) collisions
 - p, He²⁺, Be⁴⁺, C⁶⁺, Ne¹⁰⁺ - H(1s) collisions
 - p, Be⁴⁺ - H(2s, 2p₀, 2p₁) collisions
 - p - He
- Calculated all 3 types of SDCS for ionisation in p-H and p-He collisions
- Developed 1-centre method for 2-centre problems: simple, fast and works well
- Developed E1E method for ion collisions with multielectron targets
 - p - He, Li, Na, K
 - p - H₂
- The E1E results for p-He and p-H₂ agree well with experiment where available

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Igor Bray

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Thank you for attention!