Experimental Spectroscopy for Fusion Applications

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colors and plasma constituents

• plasmas show different colors with different working gas

• guessing constituents of the plasma is a kind of plasma spectroscopy

• however, color is unsuitable for quantitative analysis
spectral measurement
radiance and irradiance

sun

\[ \varepsilon(\lambda) \quad [W \text{ m}^{-2} \text{ nm}^{-1} \text{ sr}^{-1}] \]

irradiance

\[ \varepsilon'(\lambda) \quad [W \text{ m}^{-2} \text{ nm}^{-1}] \]

radiance (10^4 W m^{-2} nm^{-1} sr^{-1})

wavelength (nm)

black-body (6000 K)
• mean distance from earth

\[ L = 1.496 \times 10^{11} \text{ m} \]

• mean diameter

\[ D = 1.392 \times 10^9 \text{ m} \]

\[ \varepsilon'(\lambda) = \varepsilon(\lambda) \times \pi \left( \frac{D}{2} \right)^2 \times \frac{1}{L^2} \]

6.8 \times 10^{-5}

apparent surface area of the sun
solid angle of 1 m² area on earth
\[ \varepsilon(\lambda) \]

Radiance (\(10^4 \text{ W m}^{-2} \text{ nm}^{-1} \text{ sr}^{-1}\))

- Black-body (6000 K)

\[ \varepsilon'(\lambda) \]

Irradiance (\(\text{W m}^{-2} \text{ nm}^{-1}\))

- Irradiance on Earth

SMARTS (Simple Model of the Atmospheric Radiative Transfer of Sunshine)

http://www.nrel.gov/rredc/smarts/
7. Do the same measurements again placing the ND filter in front of the optical fiber.

8. Take background signal for each condition after turning off the lamp.

9. Derive the calibration factor $\chi(\lambda)$ with Eq. (3).

3 Observation of solar radiation

The absolute sensitivity of the measurement system has been calibrated in the previous section. Now we observe the solar radiation as an example of actual radiation source. Figure 2 shows the schematic view of the observation setup. The radiation from the entire solar surface is received with an end surface of an optical fiber. With using the calibration factor $\chi(\lambda)$ derived in the previous section, the solar irradiation $W'(\lambda)$ [Wm$^{-2}$nm$^{-1}$] on the earth can be determined.

3.1 Temperature evaluation

The visible solar spectrum consists of a broad continuum radiation and a number of absorbed lines either in the solar or the earth's atmosphere. Here we focus our interest on the macroscopic spectral profile of the continuum radiation. The measured spectral profile implies a black-body radiation, the radiance...
the solar radiation. Especially, emphasis is placed on determining the radiance on the solar surface and irradiance on the earth.

2 Sensitivity calibration

Our aim in this section is deriving the sensitivity calibration factor which can be used to derive the irradiance at the observation location. We observe a standard radiation source, for which the radiance and the radiation area is beforehand known, and derive the relation between the irradiance and the signal count number from the detector. Here, we make use of an integrating sphere as the radiation source. The inside of the sphere is coated with an agent which has high reflectance and makes complete diffusive light on reflection. The integrating sphere is equipped with a lamp inside and the emitted light from the lamp is randomly reflected on the inside wall. As a result of repeated reflections, a small aperture opened on the wall can be regarded as a homogeneous and isotropic radiation plane when observed from outside. The arrangement of the integrating sphere and optics to be used for the calibration is shown in Fig. 1. The aperture of the integrating sphere having the radiance $W(\lambda)$ and the area $s$ is observed with an end surface of an optical fiber which has the area of $a$ and is located at the distance $x$ from the radiation source. The light flux emitted from a small area $d_s$ on the radiation source is $W(\lambda)d_s$. Since the solid angle of the fiber end when observed from the radiation source is $a/x^2$, the light flux received by the fiber is $W(\lambda)(a/x^2)d_s$. By integrating this flux over the radiation source area, we obtain the total flux received by the fiber $\Gamma(\lambda)$ as

$$\Gamma(\lambda) = \int_s W(\lambda)d_s = W(\lambda)\frac{a}{x^2}$$

(1)

The irradiance at the fiber location $W'(\lambda)$ is then derived as

$$W'(\lambda) = \frac{\Gamma(\lambda)}{a} = \frac{s}{x^2}W(\lambda) \quad [W \text{ m}^{-2} \text{ nm}^{-1}]$$

(2)

When the count rate from the detector is $C(\lambda)$ [counts s$^{-1}$ nm$^{-1}$], the calibration factor $\chi(\lambda)$ is obtained as

$$\chi(\lambda) = \frac{W'(\lambda)}{C(\lambda)} \quad [J \text{ m}^{-2} \text{ counts}^{-1}]$$

(3)

By multiplying the signal count rate [counts s$^{-1}$ nm$^{-1}$] in the actual measurement with $\chi(\lambda)$, we obtain the irradiance at the observation location.

Procedure:

1. Arrange the integrating sphere and an end surface of the optical fiber as shown in Fig. 1.
2. Make sure that the observation area is larger than the aperture of the integrating sphere by introducing the laser light from the other end of the fiber (See Appendix).
3. Adjust the exposure time so that the maximum intensity signal is lower than the saturation level.
4. Take spectrum. The dynamic range of the intensity is so large that the signal in the wavelength range, e.g., $\lambda > 800$ nm, may be quite low. Therefore, the spectrum in such wavelength range should be separately measured with a longer exposure time.
5. Adjust the exposure time so that the maximum intensity signal in the $\lambda > 800$ nm range is lower than the saturation level.
6. Take spectrum.

In the actual measurement of the solar radiation, the intensity is so high that the signal may be saturated with the same optics as those used in the above procedures. In that case, we use a neutral density (ND) filter to reduce the light intensity in the whole wavelength range. Since the transmittance of the ND filter has a slight $\lambda$-dependence, the calibration must be also done with the ND filter.

Figure 1: Optical arrangement for sensitivity calibration.
irradiance (W m\(^{-2}\) nm\(^{-1}\))

wavelength (nm)

reference: [www.astm.org/g0173-03r20.html](http://www.astm.org/g0173-03r20.html)
Large Helical Device (LHD)

diameter 13.5 m
weight 1500 t
major radius 3.9 m
minor radius 0.6 m
volume $30 \text{ m}^3$
B strength 3 T
• heliotron-type device, i.e., no inductive plasma current

• advantageous for steady-state operation (no disruption)

achievements

\[ T_e \quad 20 \text{ keV} \]

\[ T_i \quad 10 \text{ keV} \]

\[ n_e \quad 10^{21} \text{ m}^{-3} \]
• spectroscopic diagnosis can be classified into two categories

• high wavelength resolution measurement
  – shift, broadening, splitting, etc.

• wide wavelength range measurement
  – intensity distribution of various emission lines, line intensity ratio, continuum
<table>
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<th>observable</th>
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<tr>
<td>shift</td>
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<td>ionizing or recombining</td>
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<td>intensity</td>
<td>$n_i$</td>
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line intensity distribution

- various emission lines are simultaneously measured
- population distribution over excited levels gives information on the plasma state
- collisional-radiative model is used for the analysis
main discharge with helium gas
The figure shows the evolution of various parameters over time (s) and wavelength (nm). The top graph plots the energy input ($W_p$) with markers for ICH, ECH, and NBI. The middle graph displays the density ($n_e$) with a peak during the gas puff (He). The bottom graph illustrates the intensity ($I(\lambda)$) for H\(\alpha\) and Hel (587.6 nm), with peaks labeled I, II, and III. The insets provide higher resolution spectra for each peak, showing the emission lines at different wavelengths.
corona equilibrium

\[ C(1, 3) n_e n(1) = [A(3, 1) + A(3, 2)] n(3) \]

more generally

\[ C(1, p) n_e n(1) = \sum_{q<p} A(p, q) n(p) \]

\[ n(p) = \frac{C(1, p) n_e}{\sum_{q<p} A(p, q)} n(1) \]
corona model cannot explain observation results
$n(p) = r_1(p)n_e n(1)$

Phase I

$T_e = 50 \text{ eV}$

$n_e = 5 \times 10^{18} \text{ m}^{-3}$
Phase III

\[ n(p) = r_0(p)n_en_i \]

HeI \( (n^3D) \)

\[ T_e = 0.4 \text{ eV} \]
\[ n_e = 8 \times 10^{18} \text{ m}^{-3} \]
\[ n_{\text{He}^+} = 1.3 \times 10^{18} \text{ m}^{-3} \]
Phase II

- recombining plasma of He II appears earlier than He I
- derived $T_e$ is higher
• spectroscopy is a fundamental diagnostic method for fusion plasmas

• collisional-radiative model is essential for analyzing measured line intensities
$T_e$ and $n_e$ analysis with helium lines

- Measurement has been made with single collimated line-of-sight
- Dominant line emission is known to be localized at edge region
• we first focus on the ratios of three emission lines which are known to have large dependence on $T_e$ or $n_e$
The diagram shows the intensity ratio as a function of line-averaged $n_e$ ($m^{-3}$) for different wavelengths. The ratios are:

- 667.8 nm / 728.1 nm
- 728.1 nm / 706.5 nm
\[ f(T_e, n_e) = \sum_p \left( \frac{n_{\text{cal}}(p) - n_{\text{mes}}(p)}{n_{\text{mes}}(p)} \right)^2 \quad \text{with} \quad p \in \{3^1S, 3^1D, 3^3S\} \]

- least-squares fitting is attempted with an error function which describes the difference between the model and measurement results
- $T_e$ and $n_e$ derived seem to be reasonable and fitting looks to be going well
obtained results are examined with using other lines
intensity (norm. by 667.8 nm)

line-averaged $n_e$ ($\text{m}^{-3}$)

Symbols: measured
Lines: synthetic

587.6 nm
388.9 nm
501.6 nm
reabsorption effectively works to reduce the transition probability of spontaneous radiative transition and to increase the upper level populations

$\Lambda(p)A(p,1^1S)$

$\Lambda(p)$ is called escape factor
\[ f(T_e, n_e, \Lambda(2^1P), \Lambda(3^1P), \ldots) = \sum_p \left( \frac{n(p) - n_{\text{mes}}(p)}{n_{\text{mes}}(p)} \right)^2 \]

- accurate evaluation of escape factors is difficult
- one idea is to include the escape factors in the fitting parameters, but restrictions should be given for the escape factors
• escape factors can be theoretically evaluated for some simplified geometries such as slab or cylindrical structures.
• restriction is added as a regularization term to the error function

\[ f(T_e, n_e, \Lambda(2^1P), \Lambda(3^1P), \ldots) = \frac{1}{N_p} \sum_p \left( \frac{n(p) - n_{mes}(p)}{n_{mes}(p)} \right)^2 + \mu \frac{1}{N_q} \sum_q \left( \frac{\Lambda(q) - \Lambda^{slab}(q)}{\Lambda^{slab}(q)} \right) \]

with \( p \in \{\text{all upper levels}\} \), \( q \in \{n^1P\} \), and hyperparameter \( \mu \)
The intensity (W m$^{-2}$ nm$^{-1}$ sr$^{-1}$) of the He I spectral lines is shown as a function of wavelength (nm). The spectrum peaks at approximately 587.6 nm, corresponding to the $^3$P$_1$ transition. The diagram on the right illustrates the energy levels and transitions for the helium ion, with vertical arrows indicating the energy differences (in eV) between the levels. The transitions highlighted in red correspond to the emission lines observed in the spectrum.
- optimizations for the hyper parameter $\mu$ and for the number of escape factors considered in the error function are attempted through bias-variance analyses
\[
(bias^2) = \frac{1}{N} \sum_{p} \left( \frac{\bar{n}_{\text{meas}}(p) - \bar{n}_{\text{fit}}(p)}{\bar{n}_{\text{meas}}(p)} \right)^2
\]

\[
\text{variance} = \frac{1}{N} \sum_{p} \left\{ \frac{1}{K} \sum_{k=1}^{K} \left( \frac{\bar{n}_{\text{fit}}(p) - n_{\text{fit}}^k(p)}{\bar{n}_{\text{fit}}(p)} \right)^2 \right\}
\]

\(K\): number of spectra taken for the same plasma condition
The calculated bias and variance of the algorithm with different $\mu$. The bias increases with an increase of the regularization parameter. It keeps relatively steady after $\mu$ is larger than 3. The variance decreases rapidly when $\mu$ is smaller than 3, and also keeps steady when $\mu$ is larger than 3. This indicates that the algorithm could be overfitted when $\mu < 3$. It is difficult to find the global minimum of the total error, but $\mu = 5$ is considered to be reasonable in the present case. It restricts the optical escape factors generated by the algorithm in a reasonable range but not completely being a function of the ground state density.

The present model could still suffer from the over-fitting problem, and we next attempt optimization for the number of escape factors considered in the model. We again conduct the bias-variance analysis with increasing the number of escape factors considered in the model. The results are shown in Figure 6.

The variance increases steadily with an increase of the number of fitting parameters. The bias decreases rapidly when the number of fitting parameters is less than 6. The total error has the minimum value when the number of fitting parameters is equal to 6. Therefore, 6 parameters ($\mu\cdot$) were selected as the fitting parameters of the algorithm.
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The variance increases steadily with an increase in the number of fitting parameters. The bias decreases rapidly when the number of fitting parameters is less than six. The minimum total error was recorded when the number of fitting parameters was six. Therefore, six parameters ($n_e$, $T_e$, $n_{e1}$, $L_{p1}$, $T_{e3}$, and $L_{p3}$) were selected as the fitting parameters for the algorithm.

4. Results and Discussion

We conducted fitting using the model described in Section 2 for the line intensity data shown in Figure 2. The electron densities and temperatures diagnosed are shown in Figure 8.

Figure 8. Electron density and temperature obtained using the novel fitting algorithms (symbols and dashed lines represent results obtained using the new model and three-line analysis, respectively). Both the electron density and temperature obtained using the novel method showed increasing and decreasing tendencies, respectively, with an increase in the line-averaged electron density, which was similar to the results obtained using the three-line method, as shown in Figure 3b. When the line-averaged electron density was low, the electron temperatures obtained using the two methods were similar. The electron temperature diagnosed using the new model decreased faster when the line-averaged electron density increased. The electron density obtained using the proposed model was generally slightly lower than that obtained using the three-line method.

A comparison of the normalized line intensities is shown in Figure 9. An example of line spectra reproduced by the new model and three-line analysis is shown in Figure 10. Compared to the three-line method, the difference between the fitted and measured results for the 706.5 and 728.1 nm lines increased slightly. A relatively large difference appeared when the line-averaged electron density was higher than $2 \times 10^{13}$ cm$^{-3}$. The fitting of the 501.6 and 587.6 nm lines improved when the line-averaged electron density was between $10^{12}$ and $2 \times 10^{13}$ cm$^{-3}$. When the line-averaged electron density was higher than $2 \times 10^{13}$ cm$^{-3}$, the 501.6 nm line had a relatively better fitting. For the 447.2 and 492.2 nm lines, the difference between the measured and fitted results improved. In general, the results fitted with the new model exhibited relatively better performance.
Conclusion

In this study, the helium CR model was revised by including the optical escape factor. The algorithm was developed to use the $E^1, E^2, E^3, E^4$ to fit eight emission lines in the visible wavelength range. According to the results, the algorithm is capable of determining the electron density and temperature of the LHD helium plasma precisely. The disagreement of the line at 501.6 nm in the conventional three-line diagnosis can be solved by including optical escape factor to the CR model and increasing the number of input lines from 3 to 8. However, in the developed algorithm, the differences between the measured and fitted results of the states $3^1S, 3^3S, 3^1D$ slightly increased compared to the conventional method. This could be improved by including statistical weight to the object function. In general, the algorithm has a good performance in the determination of the electron density and temperature.

The algorithm provides a new way to determine the electron density and temperature for the low-pressure helium plasma with OES method, and can be applied to plasma in various conditions. The validity of the algorithm for the other types of helium plasma will be further investigated in the future.
polarization spectroscopy

- anisotropy in EVDF could play a critical role for the plasma confinement
- polarization spectroscopy is a promising technique for that purpose
- polarization measurement is attempted for Lyman-α line in LHD
- anisotropy in EVDF is evaluated with a help of atomic model
Detector

Mirror at 23°

Polarization Analyzer at 68° (Brewster’s angle)

Half-waveplate

Entrance Slit

Spherical Grating
$P = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$

$n_c \text{ (m}^{-3}\text{) at } r_{\text{eff}} = 0.67 \text{ m}$
\[ P = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}} \]

quantitative analysis of polarization requires a simulation model

a sophisticated formulation has been developed by Fujimoto (Plasma Polarization Spectroscopy, 2008 Springer)
longitudinal alignment

\[ A_L = \frac{I_\pi - I_\sigma}{I_\pi + 2I_\sigma} \]

\[ A_L < 0 \quad A_L > 0 \]
under axisymmetric condition, density matrix in spherical tensor representation is written by two terms

$$\rho(p) = \rho_0^0(p) T_0^0(p) + \rho_0^2(p) T_0^2(p)$$

Population

Alignment

$a(p) / n(p)$ is related to $P$ or $A_L$
\[ Q_{0}^{0,2}(r, p) = (-1)^{J_{p} + J_{s}} \sqrt{\frac{2}{3} (2J_{p} + 1)^{-1}} \left\{ \begin{array}{ccc} J_{p} & J_{p} & 2 \\ 1 & 1 & J_{s} \end{array} \right\}^{-1} A_{L}(p, s) Q_{0}^{0,0}(r, p) \]
\[ f(v, \theta) = 2\pi \left( \frac{m}{2\pi k} \right)^{3/2} \left( \frac{1}{T^2_{\perp} T_{\parallel}} \right)^{1/2} \exp \left[ -\frac{mv^2}{2k} \left( \frac{\sin^2 \theta}{T_{\perp}} + \frac{\cos^2 \theta}{T_{\parallel}} \right) \right] \]

\[ v_{\parallel} = v \cos \theta, \quad v_{\perp} = v \sin \theta \]
\[ C^{0,0}(r, p) = \int Q^{0,0}_0(r, p) 4\pi f_0(v) v^3 \, dv \]

\[ C^{0,2}(r, p) = \int Q^{0,2}_0(r, p) \left[ 4\pi f_2(v)/5 \right] v^3 \, dv \]

\[ f(v, \theta) = \sum_K f_K(v) P_K(\cos \theta) \]

\[ f_K(v) = \frac{2K + 1}{2} \int f(v, \theta) P_K(\cos \theta) \sin \theta \, d\theta \]
Fig. 3 Simulation results for the polarization degree as a function of $T_{\parallel}$ under a fixed $T_{\perp}$ at 20 eV. The influence of the unpolarized line $S_{1/2}-2P_{1/2}$ is included.

The observation line-of-sight is assumed to be perpendicular to the quantization axis, and $P$ is evaluated as

$$P = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}},$$

where $I_{\parallel}$ and $I_{\perp}$ are the intensities of linearly polarized light components in the parallel and perpendicular directions regarding the quantization axis, respectively.

It is seen that $T_{\parallel} < T_{\perp}$ gives negative polarization degrees, i.e., $I_{\parallel} < I_{\perp}$, and the other way around with the opposite condition. It is also confirmed that the polarization is relaxed with increasing $n_e$ due to enhancement of the polarization destruction process.

3. Experimental Setup

The measurement has been made for LHD with a normal incidence VUV spectrometer having a focal length of 3 m. Figure 4 shows the line-of-sight to the plasma boundary, approximately at $r_{\text{eff}} = 0.67$ [10, 11] as shown by crossing points between the line-of-sight (horizontal dashed line) and the magnetic surface of $r_{\text{eff}} = 0.67$ (solid curve) in Fig. 4.

Figure 5 shows a schematic drawing of the spectrometer. Some optical components have been supplementarily installed in the spectrometer for the polarization measurement. The light dispersed by the grating is reflected 90 degrees into a CCD detector by two mirrors. The mirror in front of the detector is placed at Brewster's angle so that the linear polarized angle in the vertical direction is only reflected. The purpose of the second mirror is adjusting the light path angle. These two mirrors have been developed in CLASP so that the reflection efficiency is optimized at the Lyman-α line wavelength.

Another optical component is a rotatable half-wave plate placed between the entrance slit and the grating. Although the linearly polarized light in the vertical direction is always detected at the detector, the corresponding linearly polarized light in the plasma can have angles different from the vertical depending on the rotation angle of the half-wave plate. By rotating the half-wave plate during a steady-state of discharge, we can obtain linearly polarized light components at all angles as a time series.

In the actual measurement, spectra are taken every 50 ms and the rotation speed of half-wave plate is adjusted such that the angle of the linearly polarized light to be observed is rotated 22.5 degrees for every measurement in the counter-clockwise direction observed from the plasma.

Figure 6 shows an example of the discharges which the measurement has been made. The magnetic axis position and the magnetic field strength at the magnetic axis are $R_{\text{ax}} = 3.75$ m and $B_{\text{ax}} = 2.64$ T, respectively, for all the discharges.
• polarization in Lyman-α is detected for plasma of magnetically confined fusion experiment

• anisotropy in EVDF is evaluated in terms of $T_\parallel / T_\perp$ with the population-alignment collisional-radiative model

• $T_\parallel < T_\perp$ is always true, that is understandable when particle motion characteristics in the edge plasma are taken into consideration

• anisotropy shows a clear dependence on $T_e$ rather than $n_e$