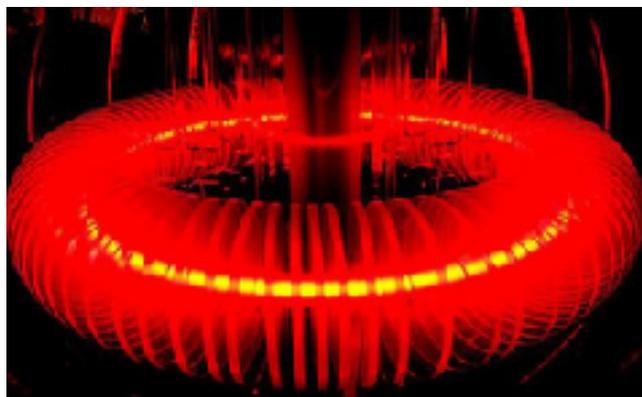
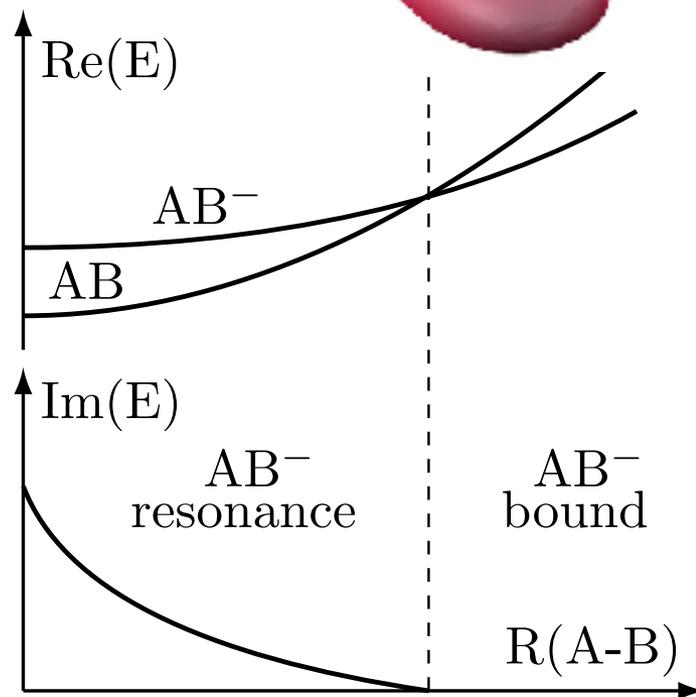
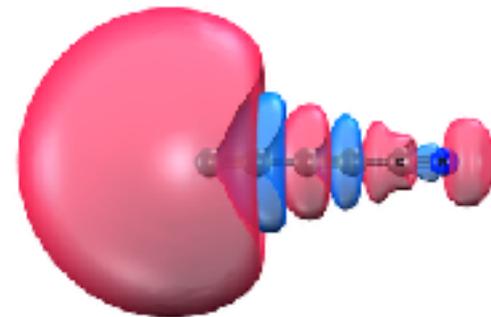


# CAP EOM-CC approach for multiple bound and continuum-embedded states: Theory and examples

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Los Angeles



ICTP-IAEA Workshop, Trieste, Italy  
April 2018

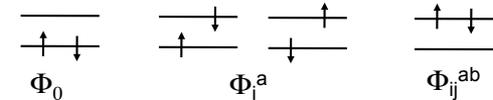




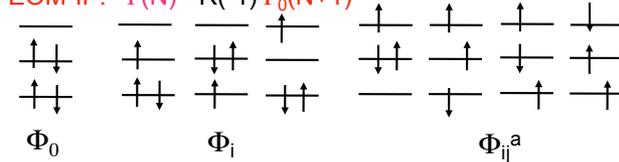
# Outline

1. Electronic resonances: What are they and why they are important in chemistry.
2. Theoretical approach: Non-hermitian QM using CAP and EOM-CC.
3. How to make sense of resonance wave functions? Extension of Dyson orbitals to metastable domain.
4. Real and imaginary excitons: Natural transition orbitals for resonance wave-functions.
5. Conclusions.

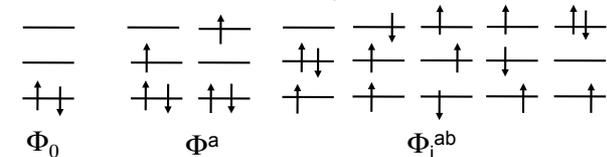
EOM-EE:  $\Psi(M_s=0) = R(M_s=0)\Psi_0(M_s=0)$



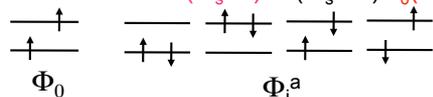
EOM-IP:  $\Psi(N) = R(-1)\Psi_0(N+1)$



EOM-EA:  $\Psi(N) = R(+1)\Psi_0(N-1)$



EOM-SF:  $\Psi(M_s=0) = R(M_s=-1)\Psi_0(M_s=1)$



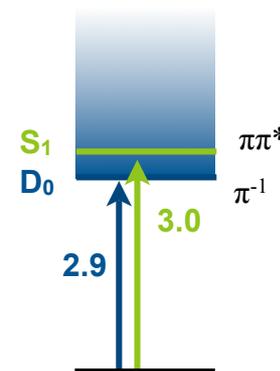
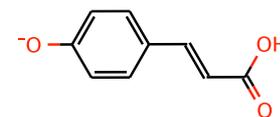
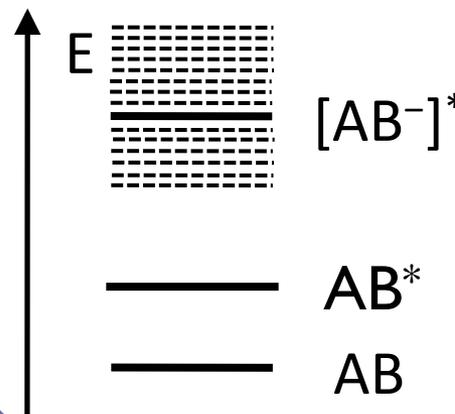
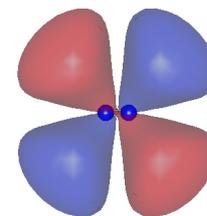
# Autoionizing states: Lie above ionization/ detachment continuum

1. Transient anions:  $N_2^-$ ,  $CO^-$ , uracil $^-$ , etc.

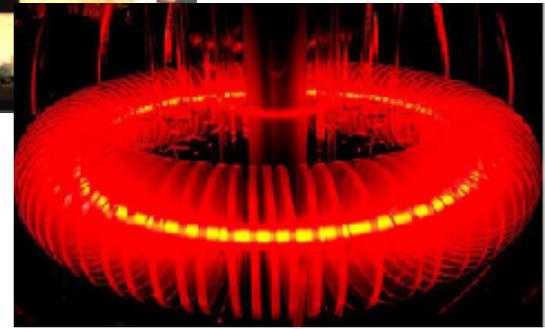
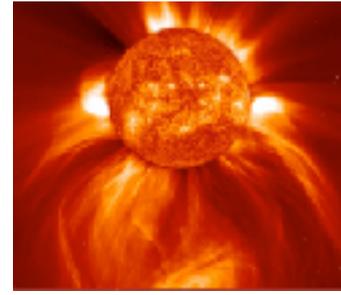
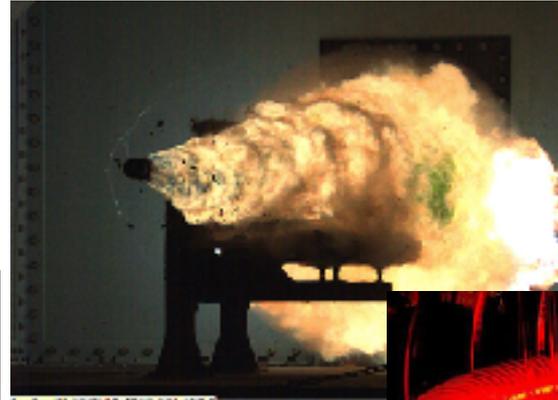
2. Excited states (including core-excited) above ionization continuum.

3. Core-ionized and doubly ionized states.

Finite lifetime, decay via auto-ionization/ detachment. Have distinct spectroscopic signatures. Their wave-functions are not  $L^2$ -integrable.



# Metastable autoionizing states in nature and technology



The visible Universe is 99.999% plasma (stars).

High energy environments (e.g., fusion reactors, plasma reactors).

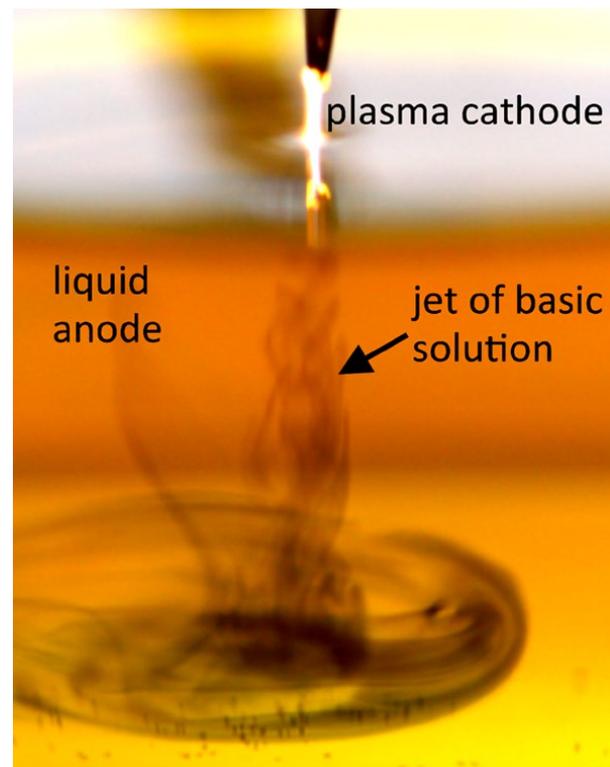
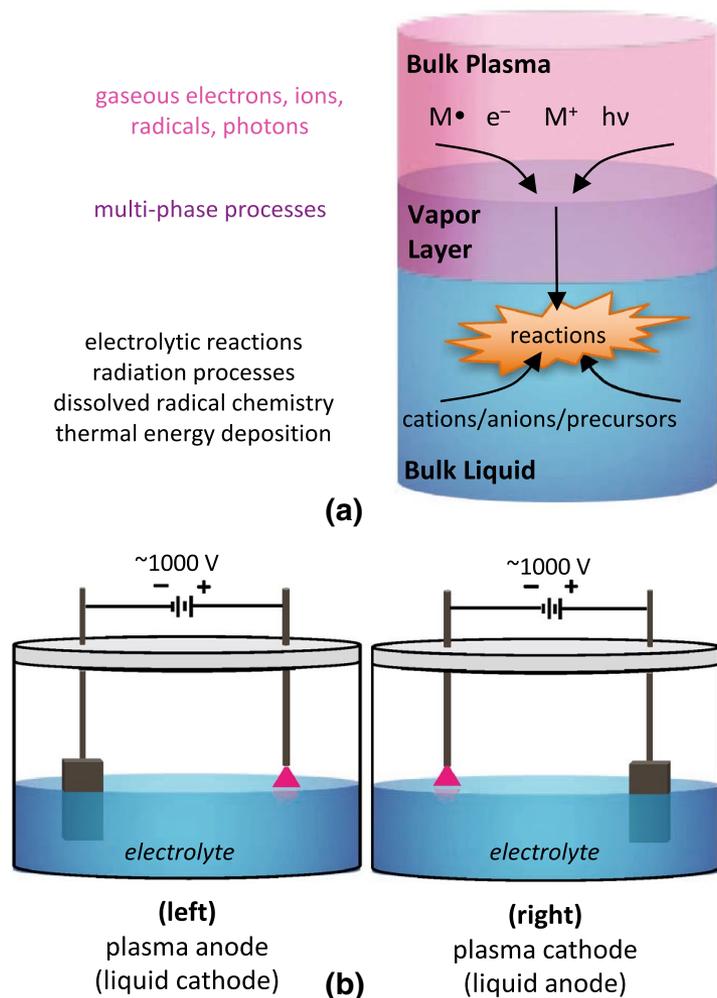
Radiolysis, DNA damage by slow electrons proceeds through metastable electron-attached states.

Interactions of molecules with metals (e.g., electrodes).

**New chemistry:** Plasmonic catalysis, plasma+solvent electrochemistry; using plasma for pollution control to remove NO<sub>x</sub> and SO<sub>x</sub> (engines, power plants).

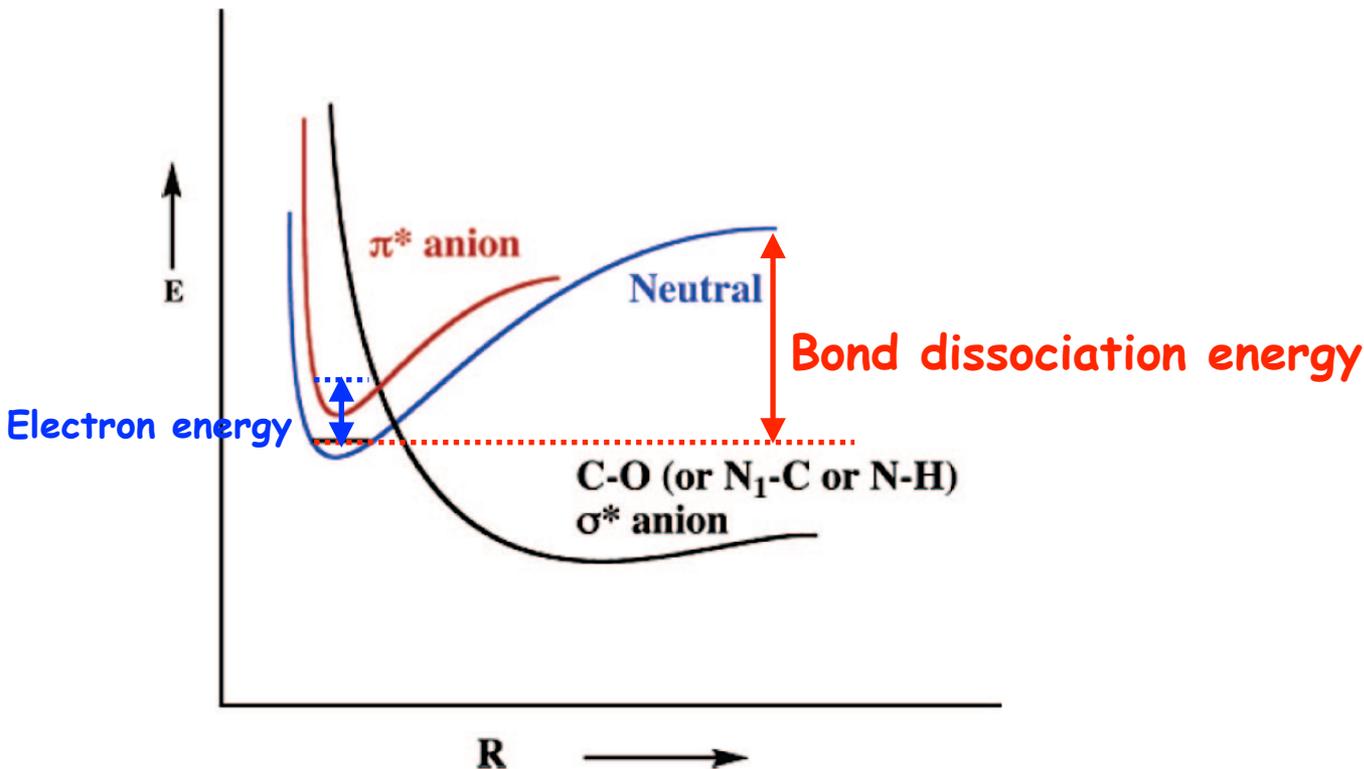
Also: new light sources and experiments.

# Plasma electrochemical cell



Rumbach and Go, Topics in Catalysis, Springer (2013)

# New chemistry facilitated by resonances via dissociative electron attachment



Electrons can break bonds and act as catalysts.  
We need new theory to develop new technologies.

Simons, *J. Phys. Chem. A*, **112**, 6401 (2008);

Herbert, *Reviews in Computational Chemistry*, **28**, 391 (2016).

# Resonances in non-Hermitian QM

Resonances can be described as discrete  $L^2$ -integrable states with complex energies

$$\psi_R(t) = \exp(-iE_R t/\hbar) \psi_R(0) \quad E_{res} \text{ resonance energy}$$

$$E_R = E_{res} - i\Gamma/2 \quad \Gamma \text{ resonance width } (\sim 1/\text{lifetime})$$

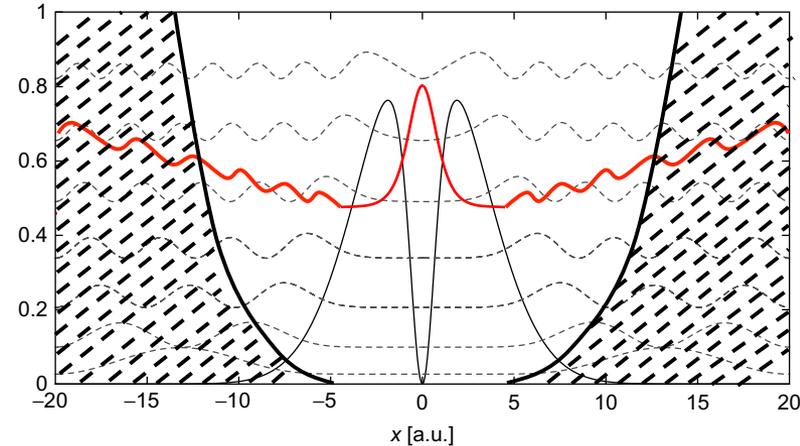
## Non-Hermitian QM:

Outgoing boundary conditions, Feshbach (diabatic) formalism, and complex-variable approaches.

**Reviews and books:** Reinhardt Ann. Rev. Phys. Chem. **33** 232 (1982); Moiseyev, Non-Hermitian quantum mechanics; Cambridge University Press, 2011.

# Complex absorbing potential (CAP) approach

$H(\eta) = H_{\text{mol}} - i\eta W(R)$ ,  
where  $W(R)$  is a box-like potential



- CAP absorbs tails of diverging wave function;
- Eigenstates of  $H(\eta)$  - complex energy of the resonance  $E_R$ ;
- $E_{\text{res}}$  depends on the strengths of the potential;
- Exact  $E_{\text{res}}$  and  $\Gamma$  are obtained in the limit  $\eta \rightarrow 0$  (in the complete basis set).

Jolicard, Austin, CPL 121 106 1985; Riss, Meyer, J. Phys. B 26, 4503 (1993) and J. Phys. B 28, 1475 (1995); Sommerfeld, Cederbaum, PRL 80 3723 (1998); Sajeev, Vysotskiy, Cederbaum, Moiseyev, JCP 131 21102 (2009).

Our impl-n: Jagau, Zuev, Bravaya, Epifanovsky, Krylov, JPCL 5, 310 (2014);  
Zuev, Jagau, Bravaya, Epifanovsky, Shao, Sundstrom, Head-Gordon, JCP 141, 024102 (2014).

# Complex absorbing potential (CAP) approach: Important issues

1. Needs to be combined with appropriate electronic structure method (multi-state, balanced, dynamical and no-dynamical correlation, size-intensive, amendable to properties, systematically improvable, correct description of the continuum onsets).

Solution: EOM-CC family of methods.

2. In finite bases, CAP is not represented exactly. Cannot go to the limit  $\eta \rightarrow 0$ . Need to work with finite  $\eta$ . This introduces the need to compute  $\eta$ -trajectories and perturbs the results.

Solution: De-perturbative correction.

# Ab initio methods for open-shell and electronically excited species

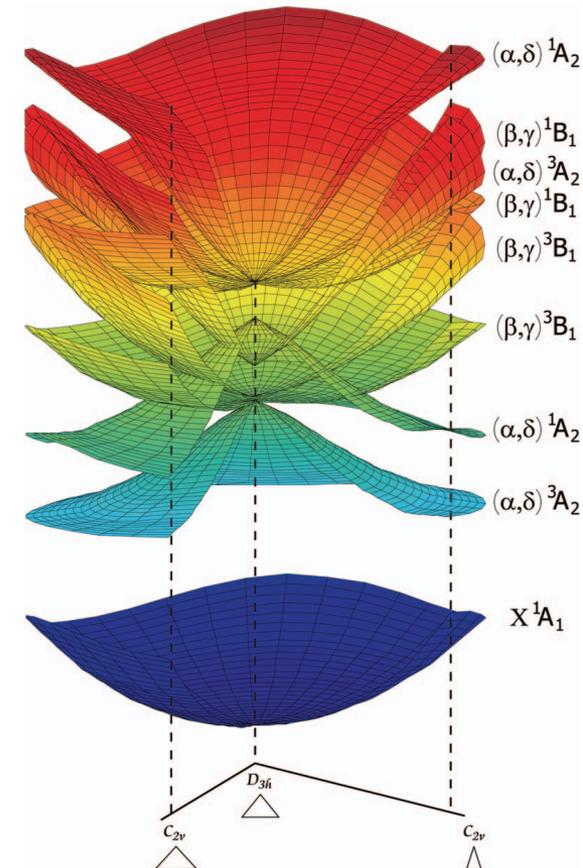
Coupled-cluster (CC) hierarchy: predictive and accurate methods for closed-shell molecules

Equation-of-motion CC (EOM-CC):  
Extends CC approach to excited states and multi-configurational wave functions

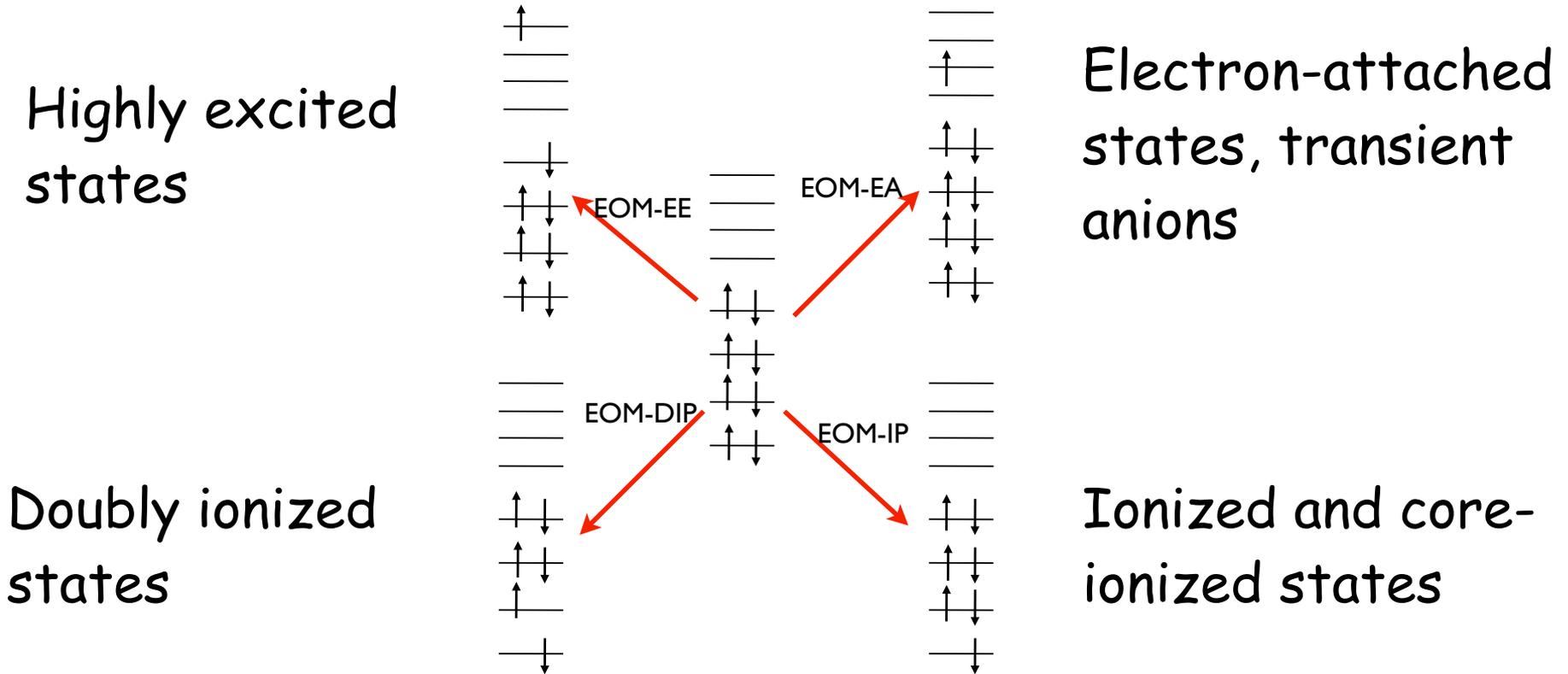
$$\Psi = R e^T \Phi_0$$

$$\overline{H} R \Phi_0 = E R \Phi_0$$

$$\overline{H} = \exp(-T) H \exp(T)$$



# EOM-CC for different types of bound states and resonances



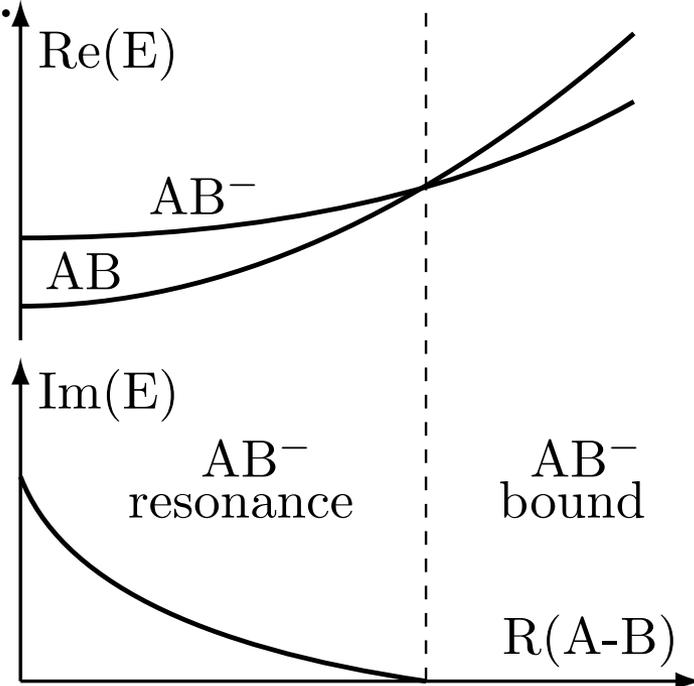
$$\overline{H}R\Phi_0 = ER\Phi_0$$

Different target stats are described by the same effective Hamiltonian

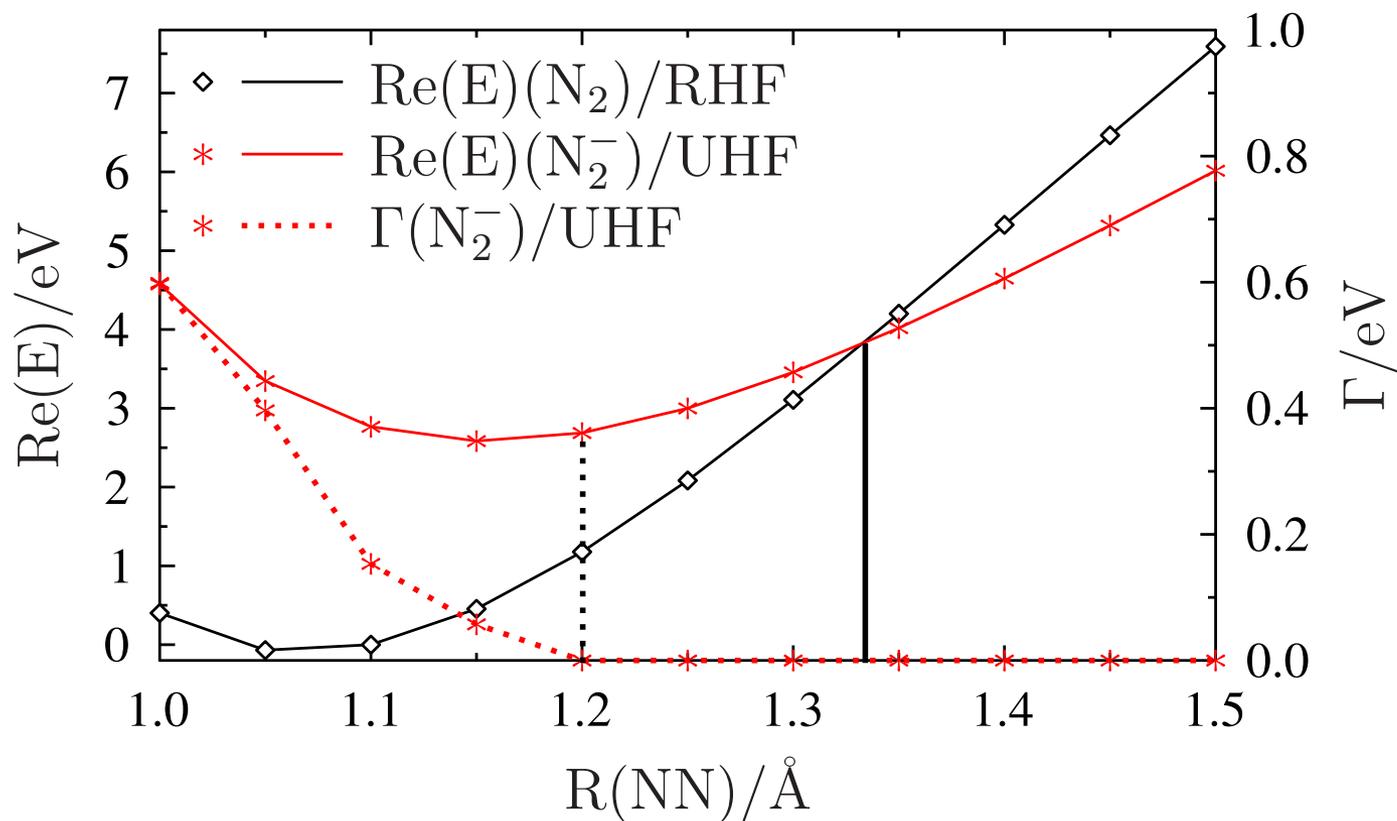
# EOM-CC for complex potential energy surfaces

Requirements:

1. PES should be smooth (both resonance positions and widths should vary smoothly upon nuclear displacements).
2. Computed energy differences (e.g., EAs) should be consistent with the computed  $\Gamma$ .

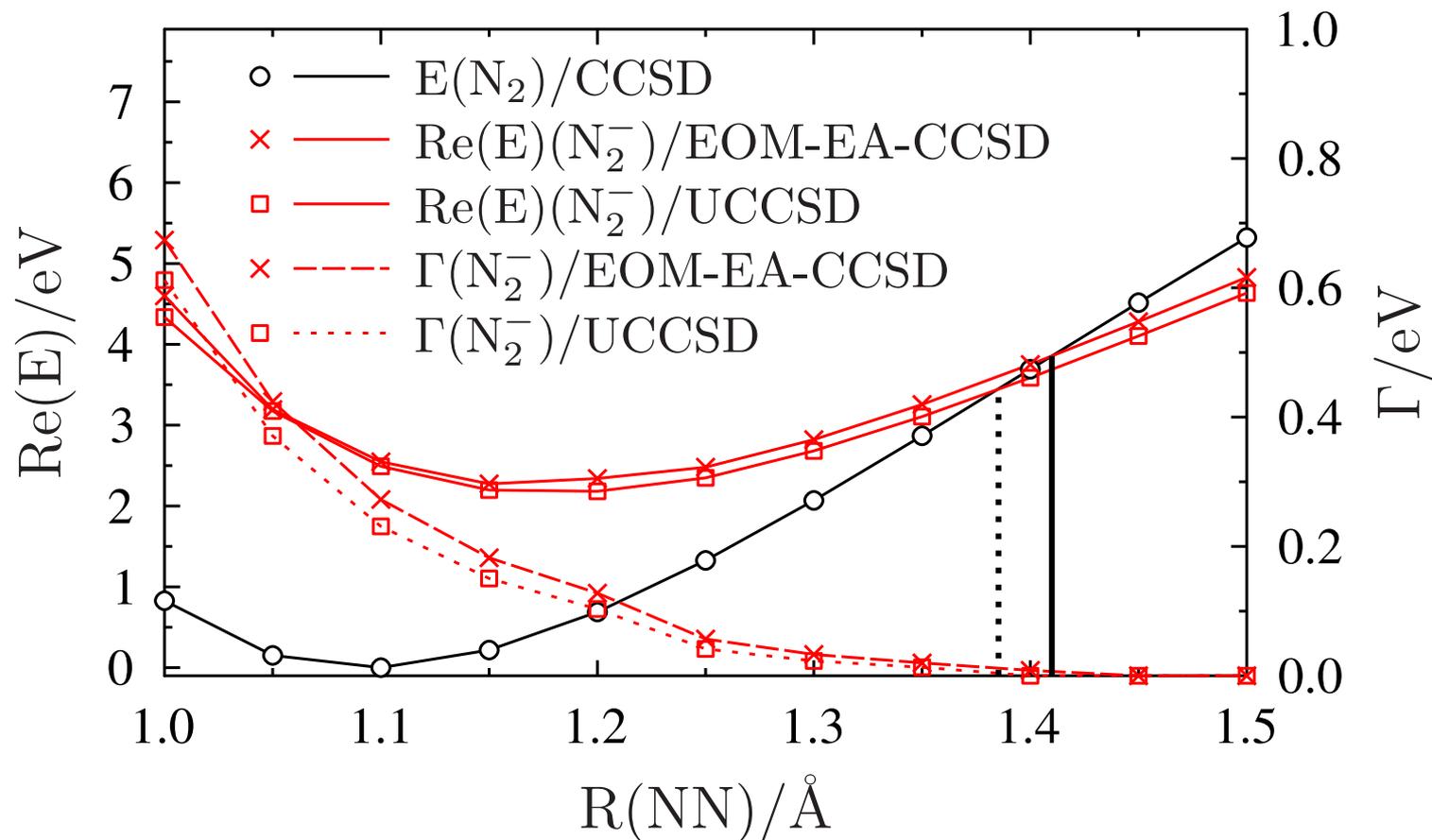


# CAP calculations of complex potential energy surfaces



CAP-Hartree-Fock PES are not internally consistent

# CAP calculations of complex potential energy surfaces

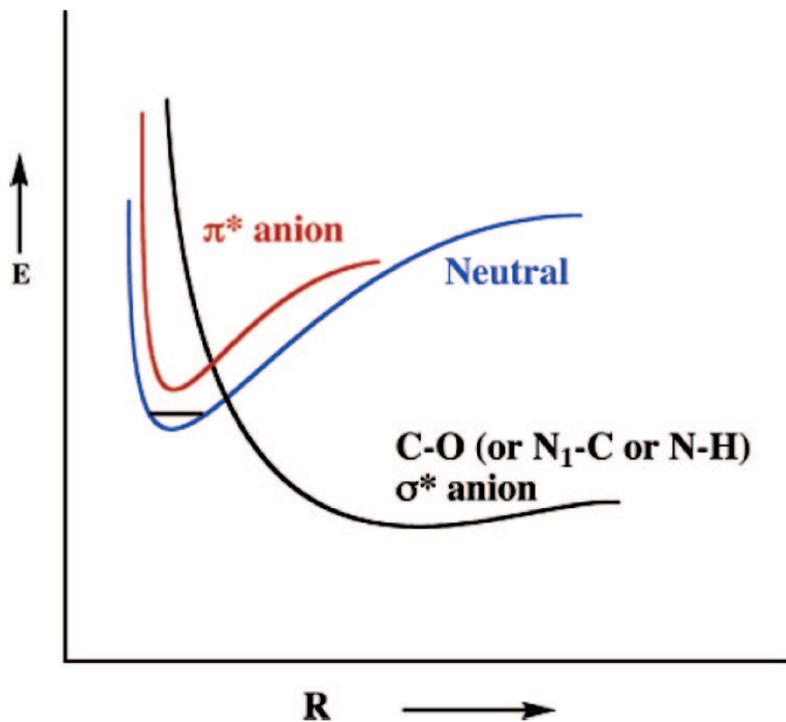


CAP-CCSD PES are less inconsistent (relative to HF)  
CAP-EOM-EA-CCSD are perfectly consistent

## Conclusions:

1. The robustness and accuracy of CAP in finite bases is improved by 1st order de-perturbative correction.
2. CAP-EOM-CC can be applied in a consistent black-box manner to molecular systems.
3. CAP-EOM-CC + de-perturbative correction: smooth PES and consistent  $dE/d\gamma$ .
4. Recent advances in Dr. Thomas Jagau group: Analytic gradients for CAP-EOM-CCSD.

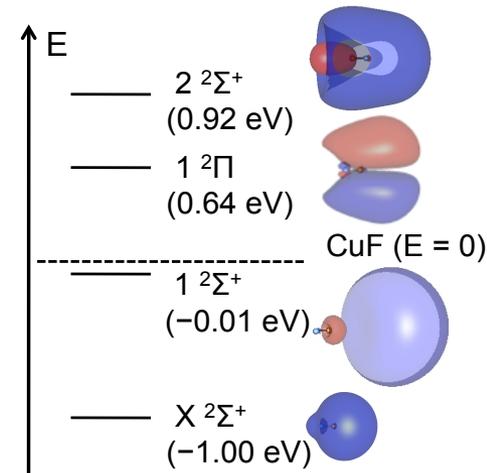
# How to make sense of resonance wave functions?



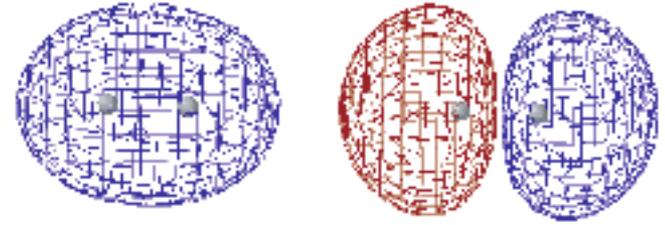
How to analyze resonance wave functions? Do molecular orbitals make sense in the domain of metastable states?

# How to analyze resonance wave functions?

1. Chemical transformations induced by electrons depend on the shape of MO to which electron is attached. **But are orbitals real?**
2. Dyson orbitals for bound states: rigorous molecular orbital picture of correlated wave functions. **Dyson orbitals are observables.**
3. Extension of Dyson orbitals to metastable states.
4. Exciton wave functions and Natural Transition Orbitals for metastable states.



# Molecular orbitals: Appear in MO-LCAO theory



One electron systems ( $H_2^+$ ):

Solving Shroedinger eqn using the basis of AOs;

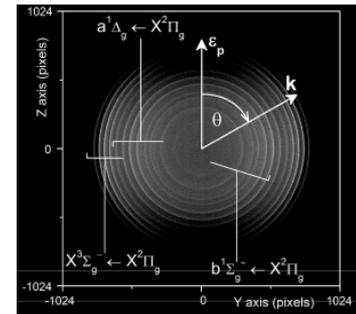
Explain bonding in molecules by electron delocalization.

Many-electron systems: MOs represent the states of pseudo-independent electrons within mean-field approximation:

$$\Phi_0 = |\phi_1 \phi_2 \dots \phi_n\rangle$$

What about correlated wave functions?

$$\Psi = \sum_K C_K \Phi_K$$



Do orbitals make sense for interacting electrons?

Are orbitals real? Do they relate to experimental observables?

# Electronic wave functions, observables, and Dyson orbitals

Initial state of N-electron system:  $\Psi_I^N(1, \dots, n)$

Final state of the ionized core (N-1 electron system):  $\Psi_F^{N-1}(1, \dots, n-1)$

Final state of ionized electron with momentum  $k$ :  $\Psi_k^{el}(n)$

Probability of ionization event (sudden approximation, dipole approximation):

$$P \sim |D_k^{IF}|^2 \quad D_k^{IF} = \mathbf{u} \langle \Psi_I^N | \mathbf{r} | \Psi_F^{N-1} \cdot \Psi_k^{el} \rangle$$

**Probability -> cross section (experimental observable).**

Using anti-symmetric properties of the wave function and one-electron nature of the dipole operator:

$$D_k^{IF} = \mathbf{u} \langle \Psi_I^N | \mathbf{r} | \Psi_F^{N-1} \cdot \Psi_k^{el} \rangle = \mathbf{u} \langle \phi^d | \mathbf{r} | \Psi_k^{el} \rangle$$

Where  $\phi_{IF}^d$  is a Dyson orbital:

$$\phi^d(1) = \sqrt{N} \int \Psi^I(1, 2, \dots, N) \Psi^F(2, \dots, N) d2 \dots dN$$

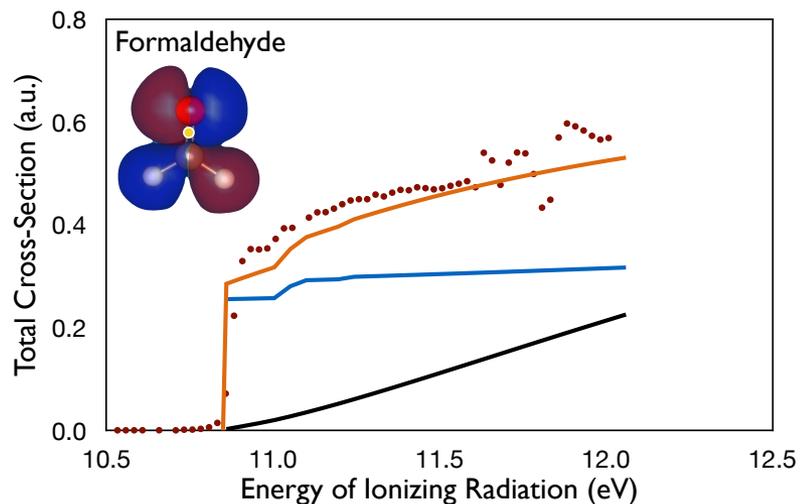
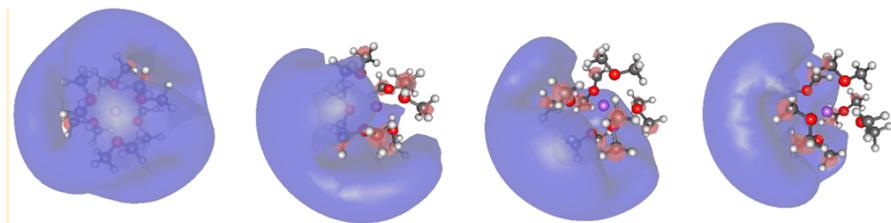
# Dyson orbitals are experimental observables

$$\phi^d(1) = \sqrt{N} \int \Psi^I(1, 2, \dots, N) \Psi^F(2, \dots, N) d2 \dots dN$$

DOs characterize the difference between N and N-1 electron states; they are generalization of Hartree-Fock MOs to general wave functions.

DOs enter the expressions of the cross-sections, they are observables.

PADs: sensitive measure of DO shape.



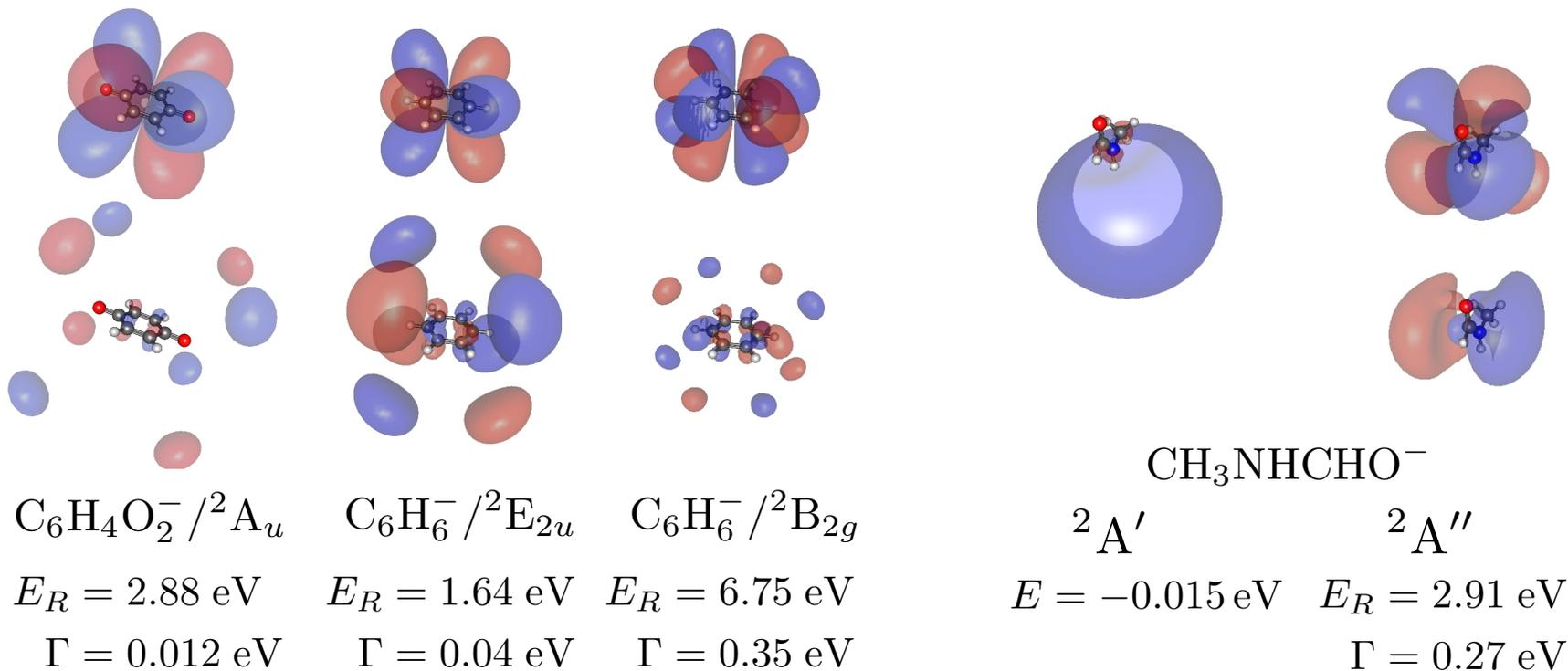
Gozem, Gunina, Ichino, Osborn, Stanton, Krylov, JPCL **6**, 4532 (2015);

Oana, Krylov, JCP **127**, 234106 (2007) and JCP **131**, 124114 (2009);

Gunina, Krylov; J. Phys. Chem. A **120**, 9841 (2016).

# Dyson orbitals in CAP EOM-CC formalism enable characterization of resonances beyond energies and lifetimes.

$$\phi^d(1) = \sqrt{N} \int \Psi^I(1, 2, \dots, N) \Psi^F(2, \dots, N) d2 \dots dN$$



Dyson orbitals for various temporary anions. The upper and lower plots show the real and imaginary parts, respectively.

# Exciton wave-functions within CAP EOM-CC formalism

Transition density matrix: reduced information about transition

$$\gamma_{pq}^{Re}(FI) = (\Psi_F^{Re} | p^\dagger q | \Psi_I^{Re}) - (\Psi_F^{Im} | p^\dagger q | \Psi_I^{Im})$$

$$\gamma_{pq}^{Im}(FI) = (\Psi_F^{Im} | p^\dagger q | \Psi_I^{Re}) + (\Psi_F^{Re} | p^\dagger q | \Psi_I^{Im})$$

One-electron mapping between initial and final states. For bound initial state:

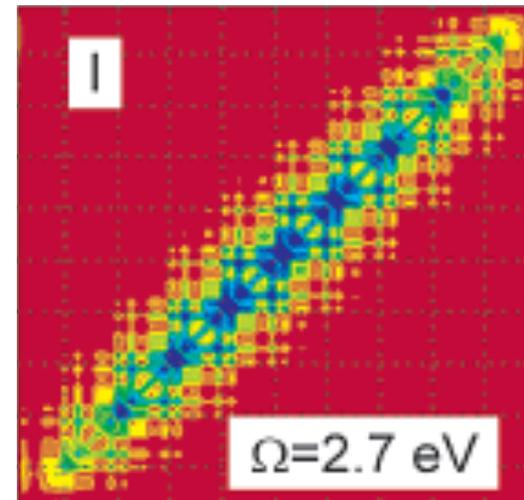
$$\Psi_F^{Re} = \sum_{pq} \gamma_{pq}^{Re} p^\dagger q \Psi_I + \text{higher excitations}$$

$$\Psi_F^{Im} = \sum_{pq} \gamma_{pq}^{Im} p^\dagger q \Psi_I + \text{higher excitations}$$

Transition density represents exciton wave-function

$$\xi(r_p, r_h) = \sum_{pq} \gamma_{pq}(FI) \phi_p(r_p) \phi_q(r_h)$$

Transition density related to observables



# Natural Transition Orbitals within CAP EOM-CC

Natural Transition Orbitals: Diagonal representation of the transition OPDM (SVD of gamma):

$$\xi(r_p, r_h) = \sum_{pq} \gamma_{pq} \phi_p(r_p) \phi_q(r_h) = \sum_K \sigma_K \tilde{\phi}_K^p(r_p) \tilde{\phi}_K^h(r_h)$$

Or give most compact representation of the transition:

$$\Psi_F = \sum_K \sigma_K a_K^\dagger i_K \Psi_I + \text{higher excitations}$$

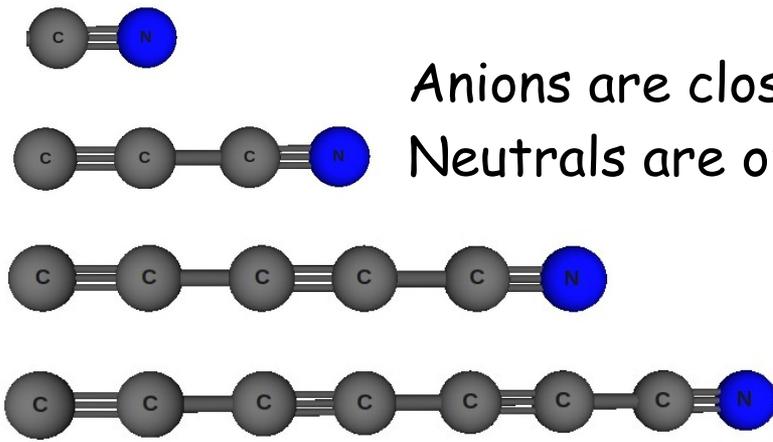
And are related to the observables:

$$\langle \Psi_F | \mu | \Psi_I \rangle = \text{Tr}[\gamma \mu] = \sum_K \sigma_K \langle \tilde{\phi}_K^h | \mu | \tilde{\phi}_K^p \rangle$$

Resonance wave-functions: Real and imaginary excitons

Luzanov, Zhikol, In Practical aspects of computational chemistry Springer, 2012;  
Plasser, Wormit, Dreuw, JCP **141** 024106 (2014); Bappler, Plasser, Wormit,  
Dreuw PRA **90** 052521 (2014); Head-Gordon et al, JPC **99** 14261 (1995);  
Martin, JCP **118** 4775 (2003).

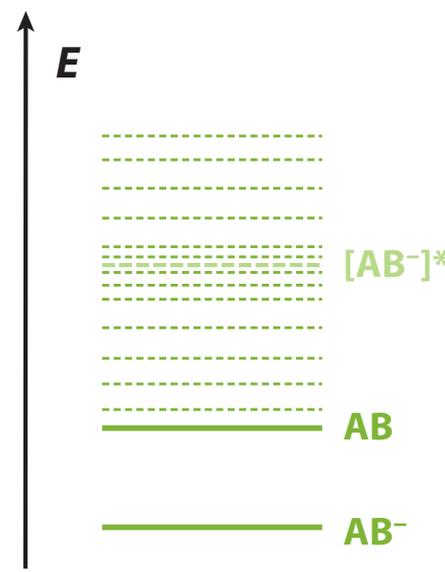
# Example: Bound and metastable states in cyanopolyynes $C_{3x}N^-$



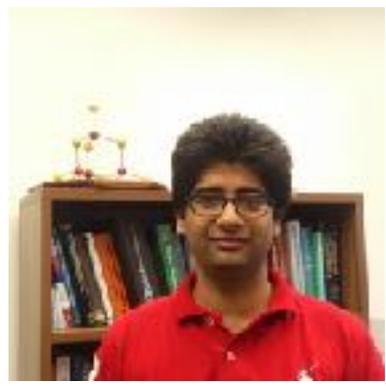
Anions are closed-shell species (singlets)  
 Neutrals are open-shells (doublets)



Dr. Wojtek Skomorowski

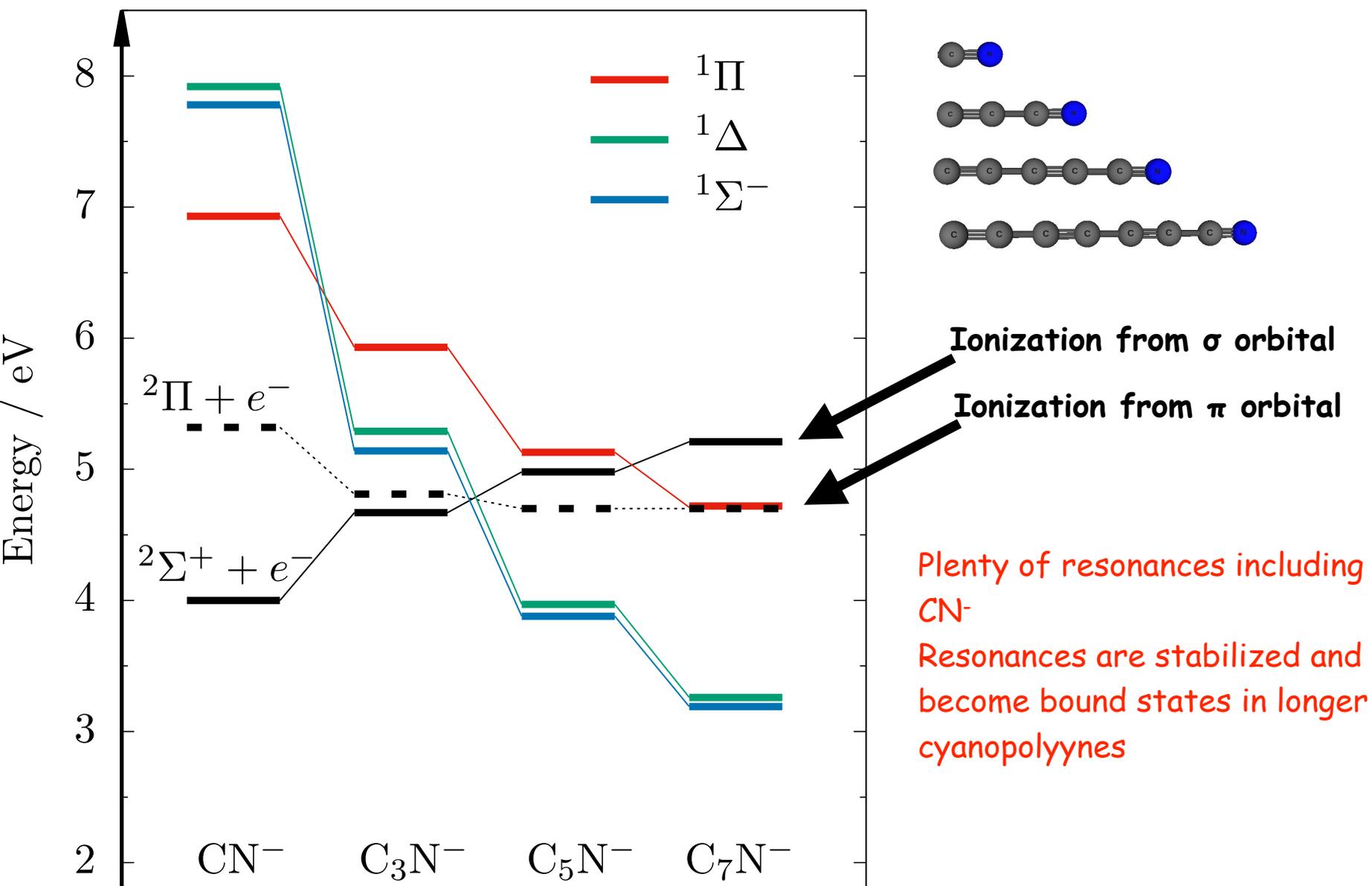


Studied by Neumark (exp), Simons, Sommerfeld, Crawford (theory), and others.  
 Our work: Skomorowski, Gulania, Krylov, PCCP **20**, 4805 (2018).

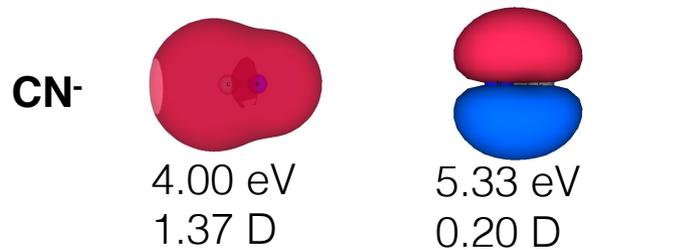


Sahil Gulania

# Resonances and bound states in cyanopolyynes

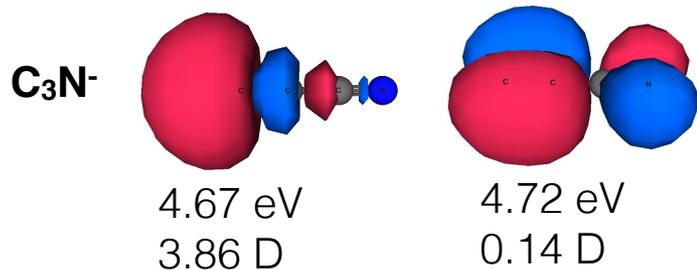


# Electron-detached states of the $C_nN$ anions

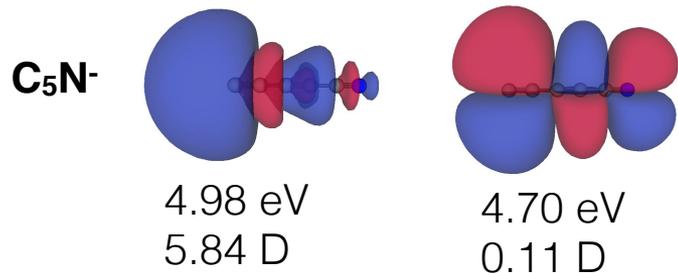


Large electron affinities/detachment energies;

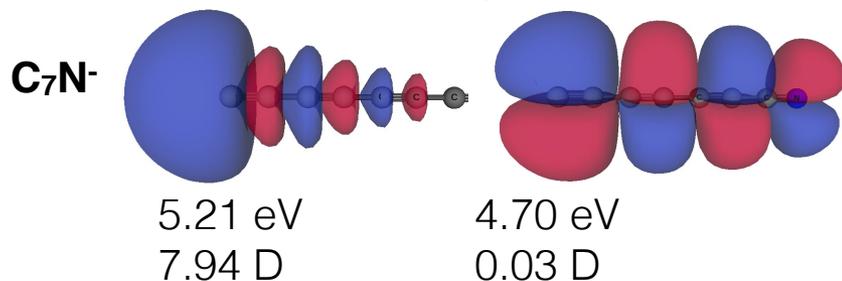
Detachment from  $\sigma$  orbital leads to large dipole moment;



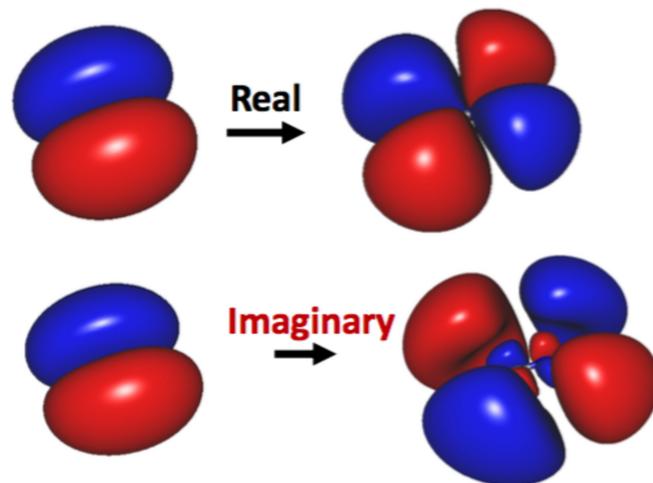
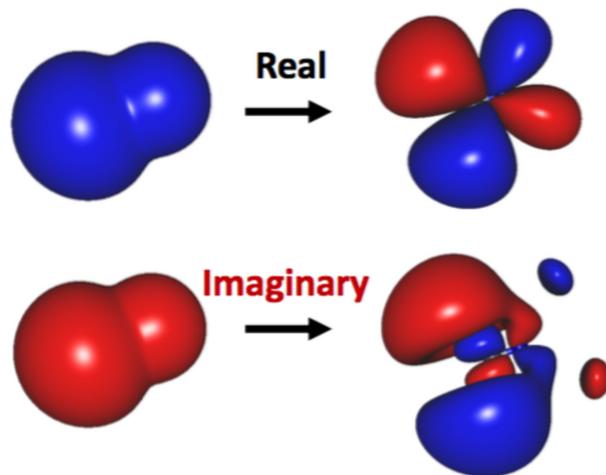
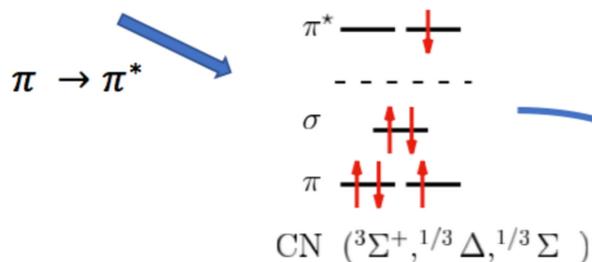
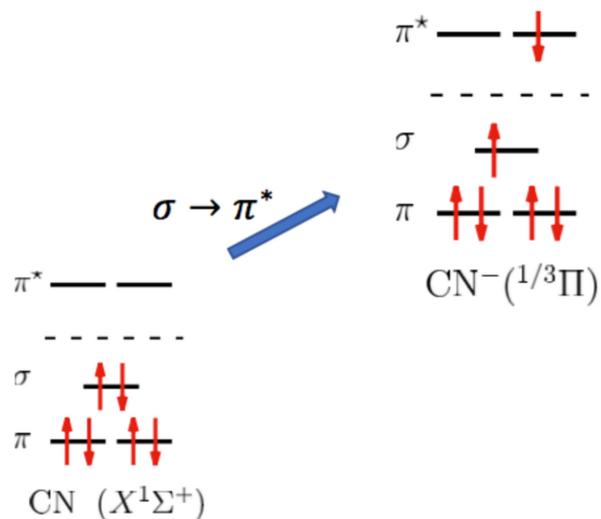
Detachment from  $\pi$  orbital: small dipole moment;



$\sigma$  orbital is stabilized in longer chains by increased dipole moment

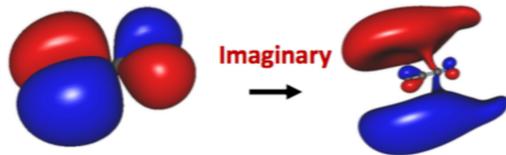
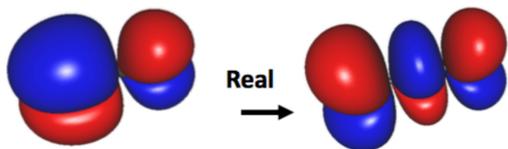
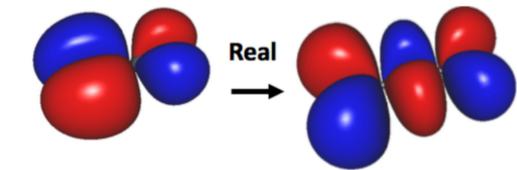


# Example: NTOs for resonances in $CN^-$ (more in W. Skomorowski talk/poster)

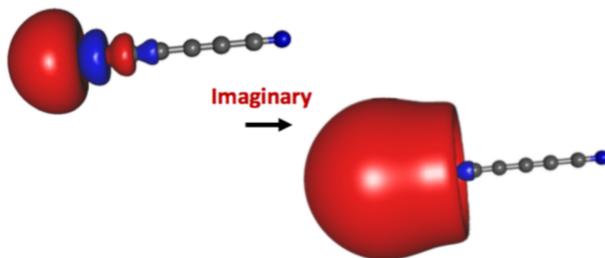
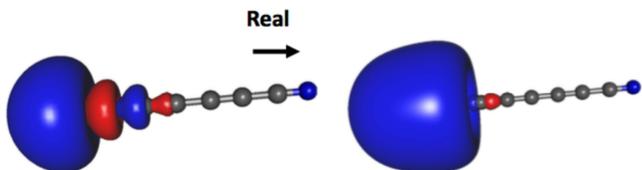
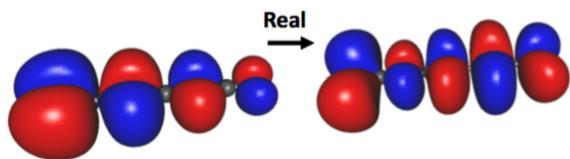
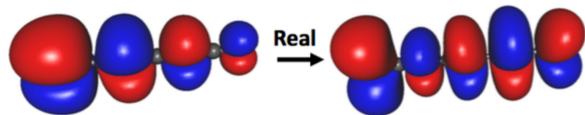
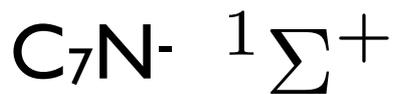


Hole orbital: Same for real and imaginary part of gamma  
 Particle orbital: very different!

# Real and imaginary excitons (more in W. Skomorowski talk/poster)



$\text{PR}_{\text{NTO}}=2$

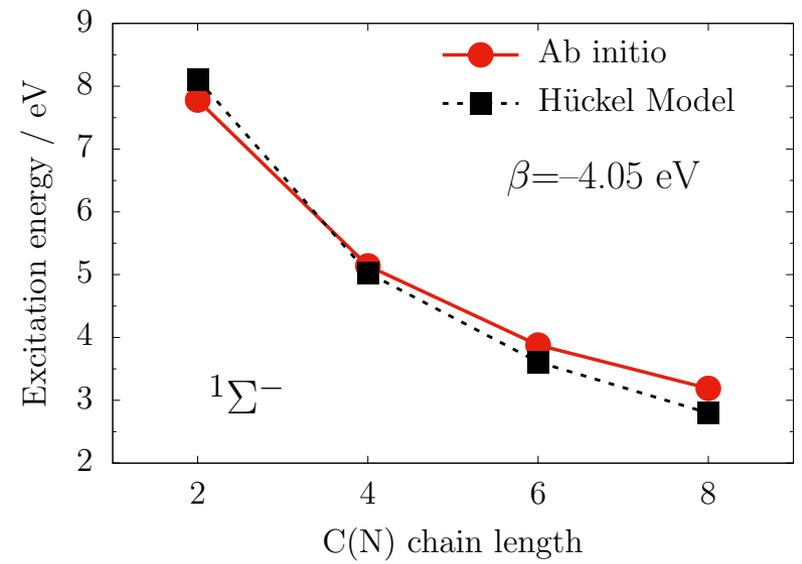
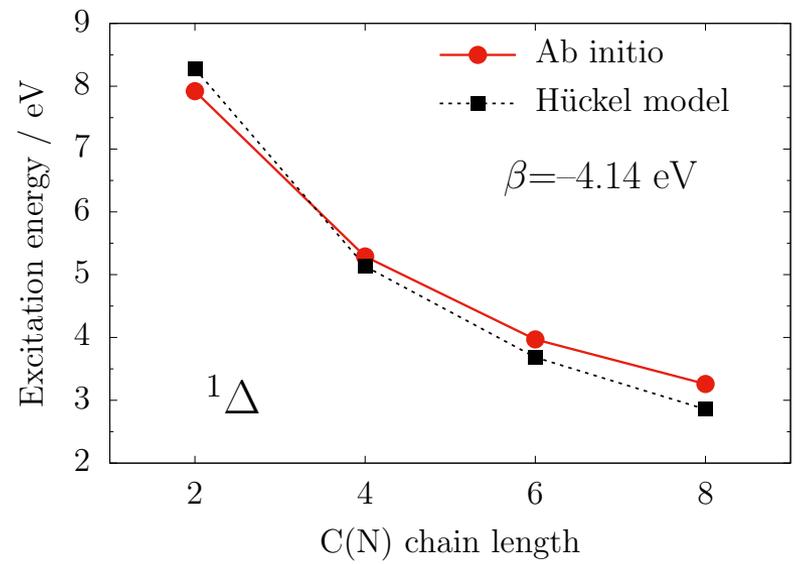
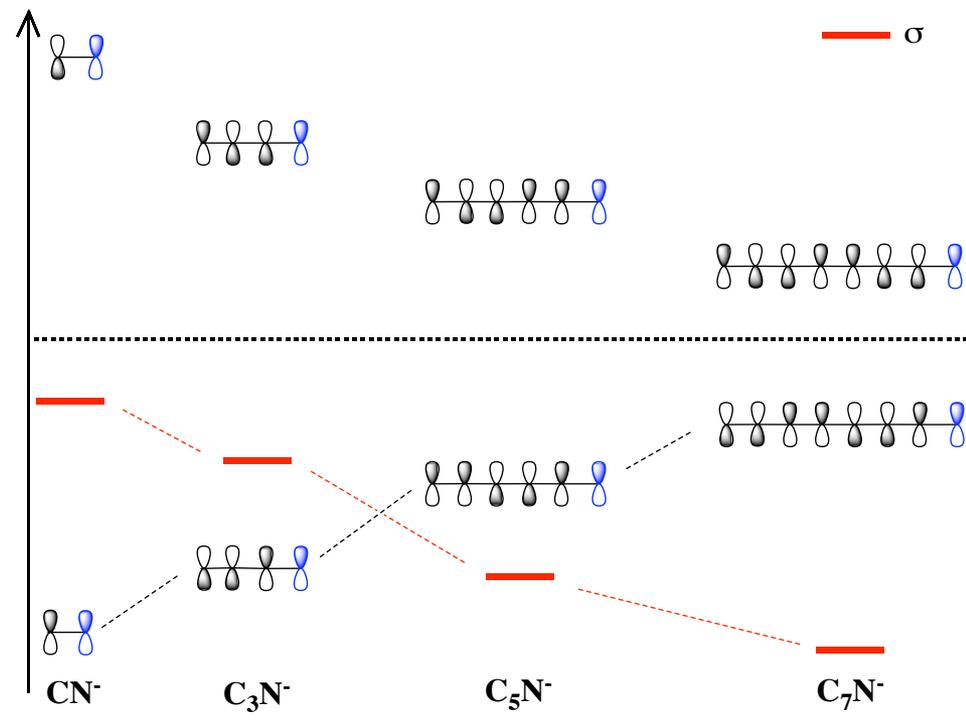


$\text{PR}_{\text{NTO}}(\text{Re})=2.9$

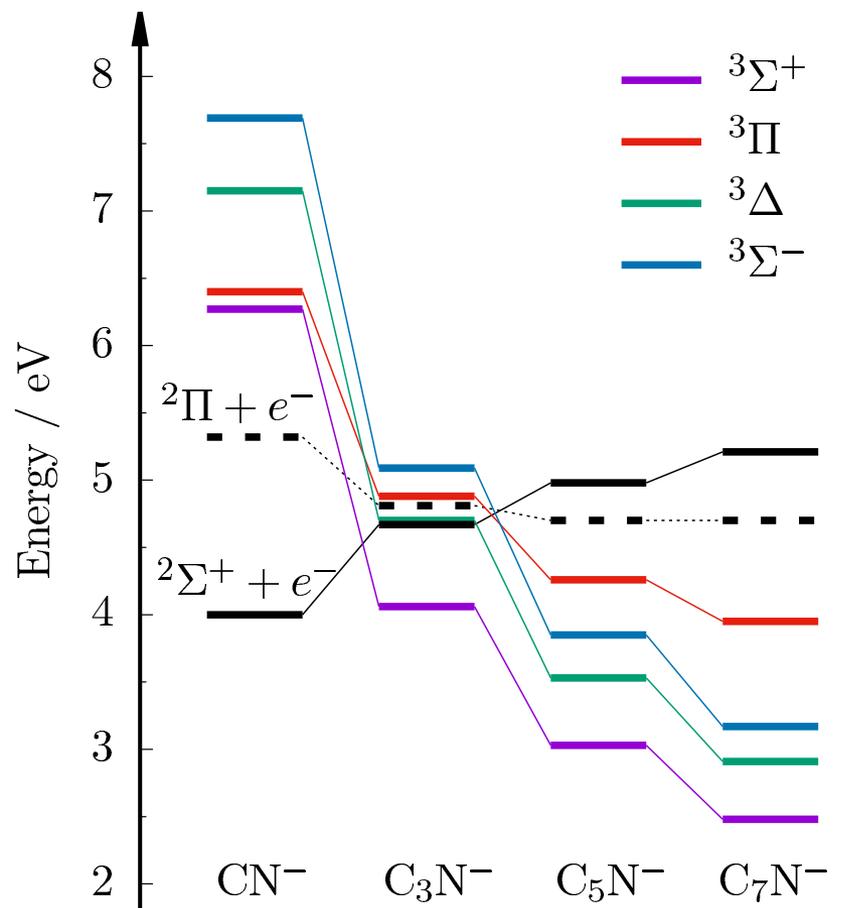
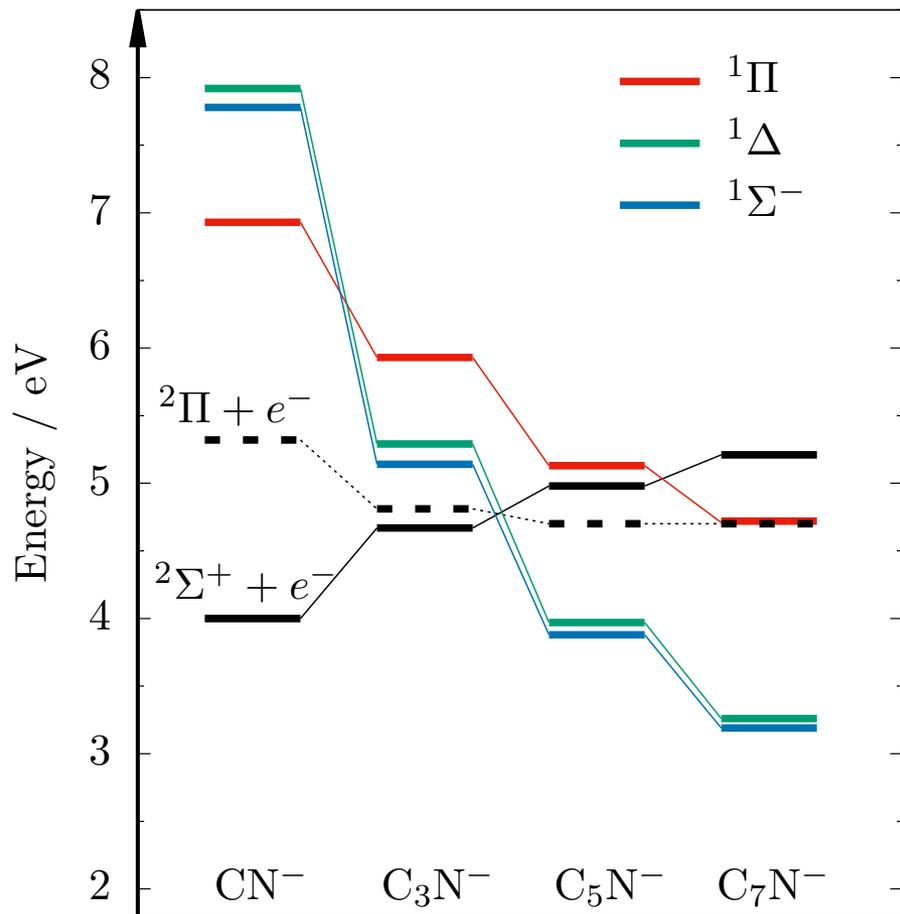
$\text{PR}_{\text{NTO}}(\text{Im})=1.4$

# Huckel model explains trends in resonances and bound states in cyanopolyyenes

$$E_{ex}(\pi\pi^*) = -\beta \sin \frac{\pi}{2(1+N)}$$



# Resonances and bound states in cyanopolyyenes



Trends in all  $\pi\pi^*$  states can be fit to Huckel model with  $\beta=3.2-4.1$

# Conclusions

1. Resonances offer new opportunities for chemistry: New reaction pathways via electron-attached states.
2. Non-Hermitian QM: EOM-CC + CAP approaches: robust and uniform framework for bound and unbound states of different types (super-excited states, transient anions, core-level states).
3. Opportunity to test and extend important chemical concepts into a new domain. Examples: Dyson orbitals, Natural Transition Orbitals, Huckel theory, and more.

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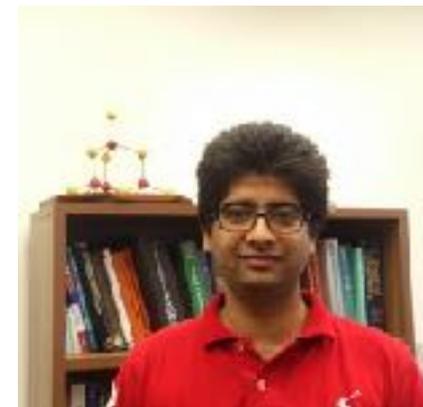
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