

**Atomic Processes in Plasmas,  
IAEA, 15-19 May 2023**



# Tracking of internal states in collisional-radiative models employed in the transport codes

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J.Romazanov, M.Groth, P.Boerner,  
S.Brezinsek, Y.Marandet**

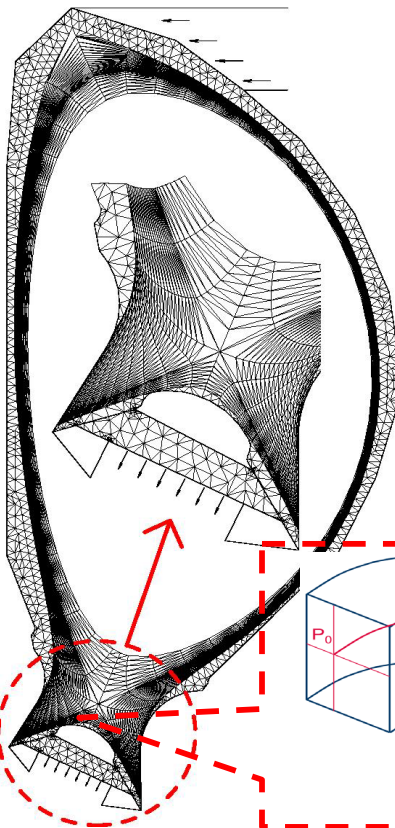


This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.



# **Monte-Carlo edge plasma transport codes: EIRENE and ERO**

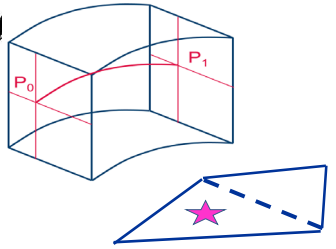
# What is EIRENE in a nutshell (e.g. in SOLPS)?..



Macroscopic:

**CFD codes**  
(computational **fluid**  
dynamic):  
B2, Edge2D, EMC3,  
SOLEdge3X, etc...

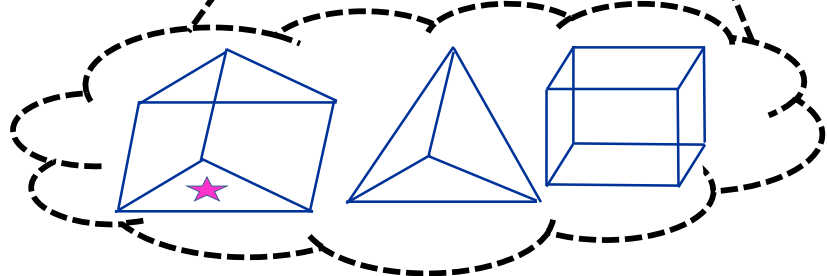
2D or 3D Volume grid  
(e.g. tetraeder) adopted  
for current magnetic  
configuration



Plasma flow parameters

**3D EIRENE  
volume cell**

**Source terms (Particle,  
Momentum, Energy)**

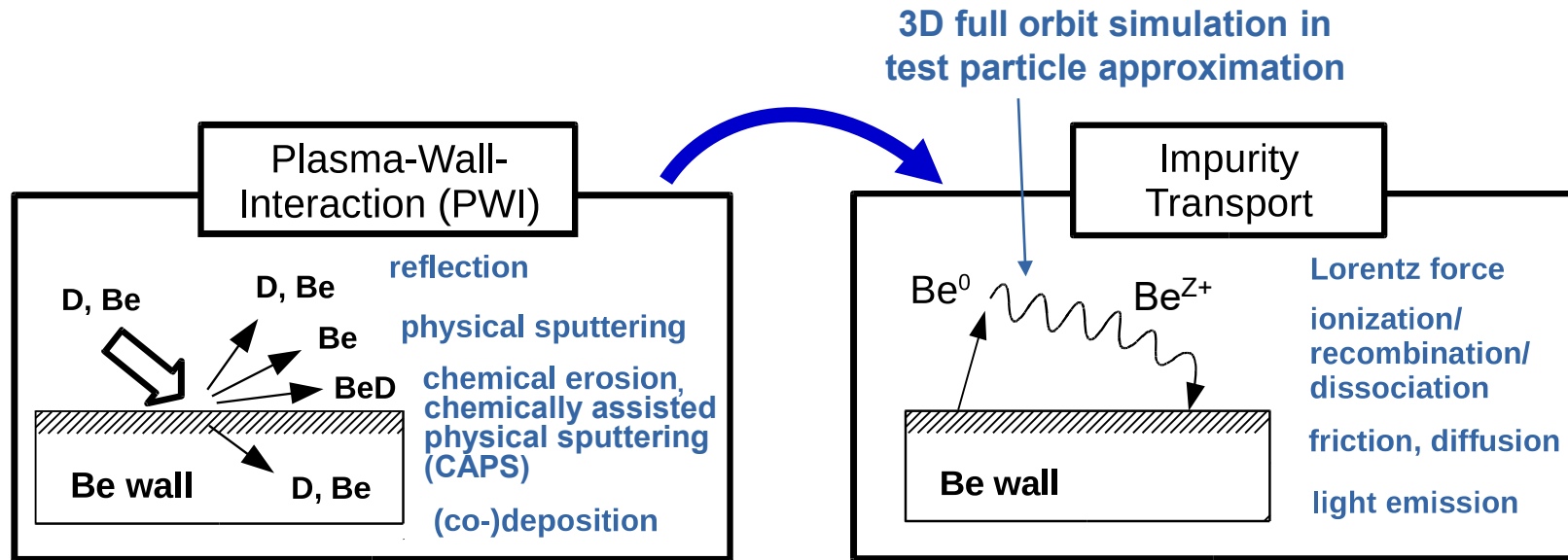


Microscopic:

**EIRENE-NGM:** a  
3D3v MC multi-  
species transport  
code incl. radiation  
transfer, **kinetic**  
or **F-K** hybrid.

**CRMs**  
(HYDKIN, AMJUEL,  
ADAS, ...)  
for atomic and  
molecular neutrals  
 $H, H^*, H_2, H_2(v), H_2^*, H,$   
 $H_2^+, \dots$ , impurities  
ionisation, CX,  
recombination etc.

# ERO code: concept and design



**Significant code upgrade ERO2.0:**  
**Based on same principles, but allows extending the simulation volume significantly (using HPC)**

# Transport code variations and commons: CRMs

Property	EIRENE	ERO, ERO2.0
<b>Ab initio</b>	<b>Yes</b> (iterations with fluid code, e.g. with B2.5 → “SOLPS-ITER”)	<b>No</b> , test-particle approximation on a given plasma BG (e.g. from SOLPS)
<b>Gyro-motion</b>	Guiding centre approximation	<b>Full-resolved</b> , detailed 3D wall elements, <b>surface sheath</b>
<b>Plasma-wall interaction</b>	Less detailed locally, often <b>no 3D</b>	<b>More detailed</b> : 3D, sheath, etc.
<b>PWI impact on plasma</b>	<b>Yes</b> , also “advanced fluid neutrals”, “wall reservoirs” etc.	In general <b>NO</b> (certain effects were introduced...)

**Both approaches have clear advantages and range of appropriate tasks, both packages are highly demanded by ITER, EU-DEMO and other fusion-relevant devices!**

*→ Moreover, they often support each other (plasma BGs from various CFD-EIRENE used and tested in ERO)*

**However, the subtasks for treatment of atomic and molecular processes in plasma are very similar!**

- Break-up of the molecular species in plasma
- Ionisation and other processes affecting charge and energy, thus transport
- Synthetic spectroscopy for comparison with diagnostics

**Similar A&M process treatment challenges!**



## **Collisional-radiative models (CRMs):**

- treating none-stationary**
- metastables (MS) in atoms**
- vibrational states in molecules**



CRM (technically) = reaction data + SOLVER

❑ Most solvers solve algebraic **stationary** system of equations  $\frac{dN_i}{dt} = 0$

➔ in such cases the solver is much less valuable than the A&M data collection

➔ this statement is underlined even by the developers of the very well established and mostly up-to-date YACORA model

D.Wünderlich and U.Fantz,  
*Atoms* **2016**, 4(4), 26

❑ CRM containing molecular species is even more complex, e.g. one needs to track more processes and more states (vibrational, rotational).

➔ Tracking all the states as separate species demands enormous CPU and memory resources.

❑ EIRENE pre-calculates rates for each volume cell. It also allows to pre-calculate values for various  $T_i/T_e$  ratios.

# MS-effect for Be ("generalised" MS $10^{-4}$ s)

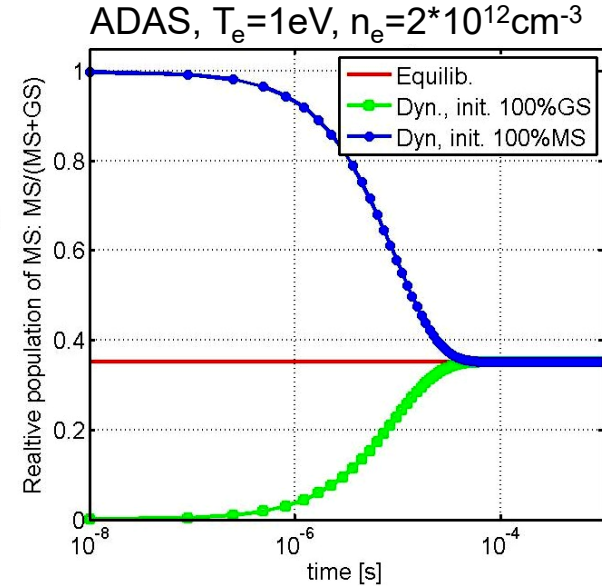
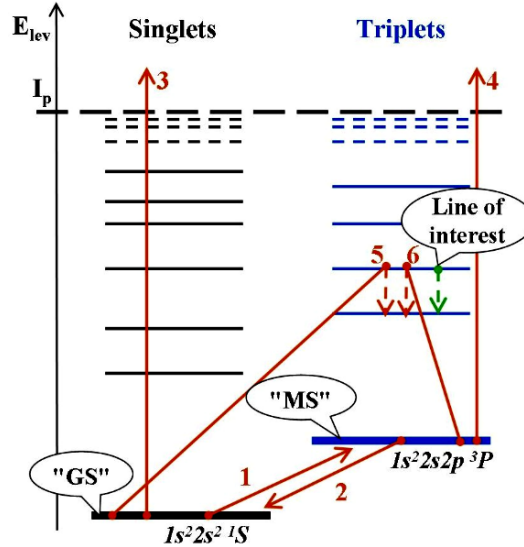


The system of 2 balance equations can be solved analytically . . .

$$\begin{cases} \frac{dN_{GS}}{dt} = -\langle ExGM \rangle N_{GS} - \langle IzG \rangle N_{GS} + \langle ExMG \rangle N_{MS} \\ \frac{dN_{MS}}{dt} = -\langle ExMG \rangle N_{MS} - \langle IzM \rangle N_{MS} + \langle ExGM \rangle N_{GS} \end{cases}$$

( $C_{1i}$ ,  $C_{2i}$ ,  $\lambda_p$ ,  $\lambda_m$  determined by rates):

$$dN_i(t) = C_{1i} \exp(-\lambda_p t) + C_{2i} \exp(-\lambda_m t)$$



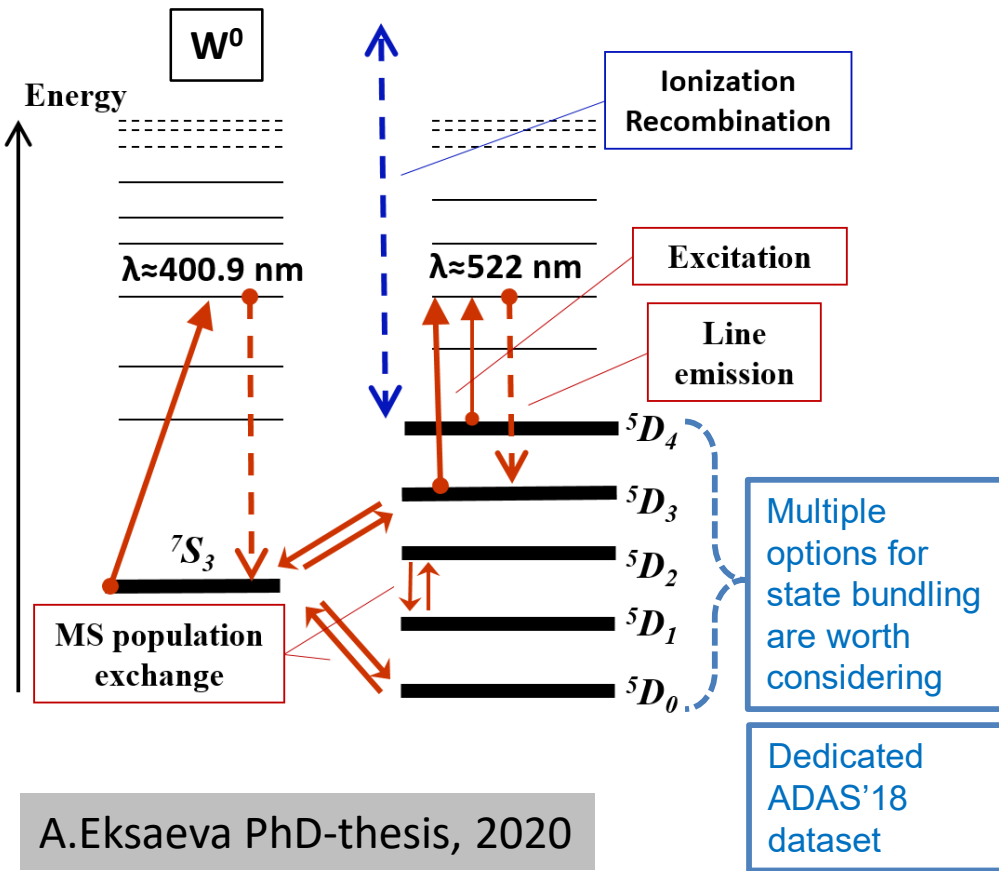
## Effective rates:

- 1,2) transitions  $\langle Ex.. \rangle$  between "GS" and "MS"
- 3,4) ionizations  $\langle Iz.. \rangle$  from "GS" and "MS"
- 5,6) line intensity (PEC – photon emission coefficient), contributions from "GS" and "MS"

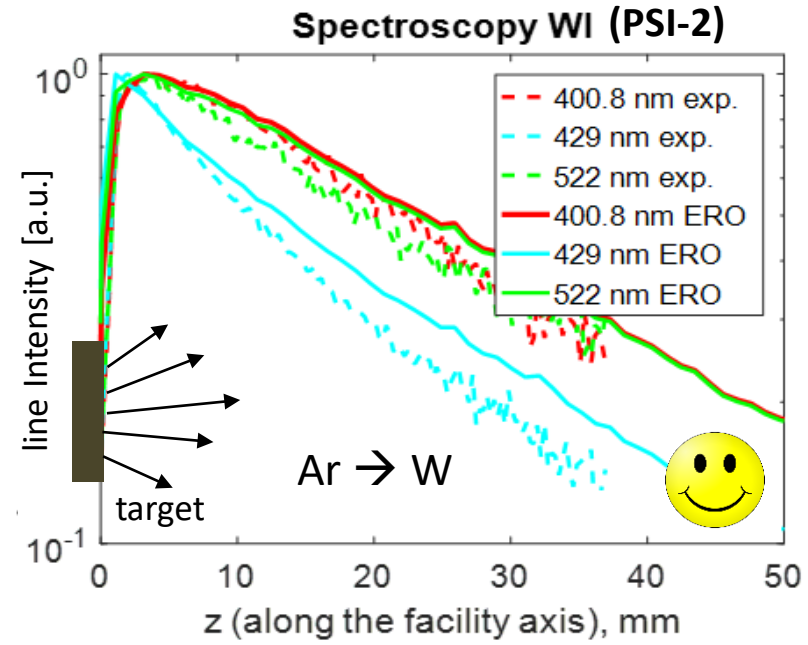
- ❑ MS-population tracking was introduced into ERO and demonstrated to be of significance by applications to PISCES-B, PSI-2 and JET-ILW (**neutral Be, He, W**)
- ❑ For WI dedicated ADAS dataset was developed; various lower states bundling options ( $\sim 10$ ) tested etc.



# MS-effect for WI (PSI-2 linear plasma device, JET)

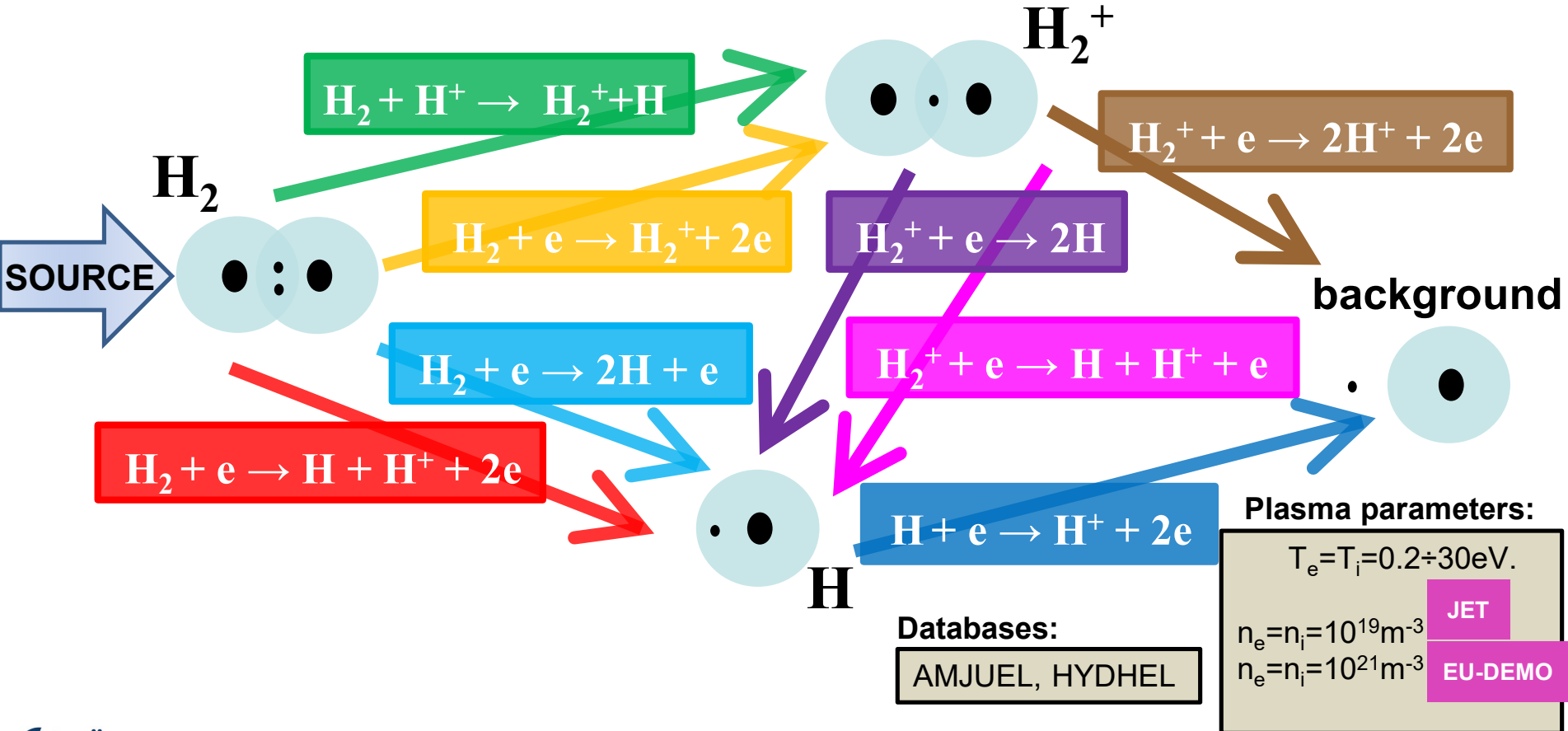


A.Eksaeva PhD-thesis, 2020

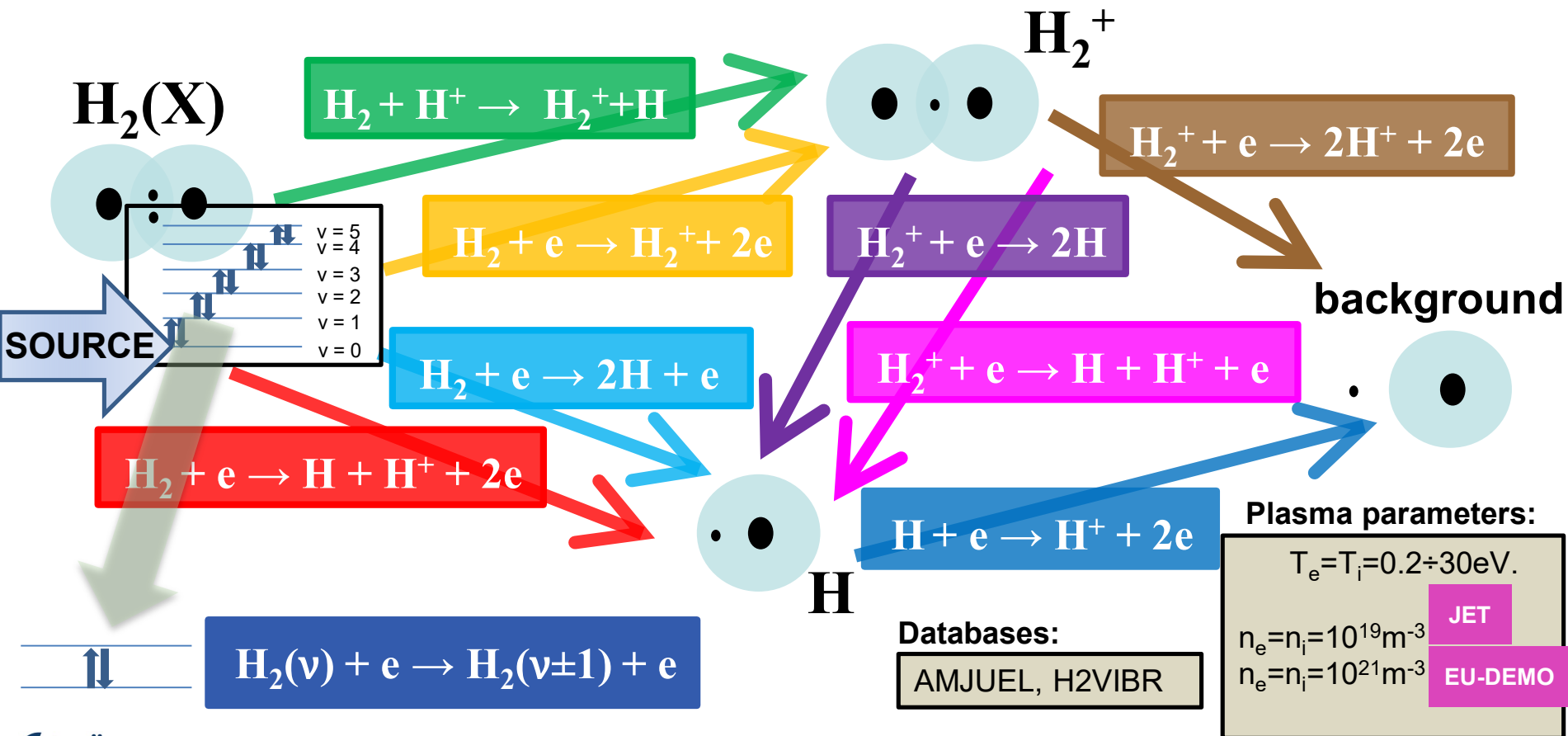


✓ **Test in tokamak (JET ITER-like Wall): MS tracking reduces simulated line intensity by x1.7!**

# CRM FOR MOLECULES



# CRM FOR MOLECULES : vibrational resolution

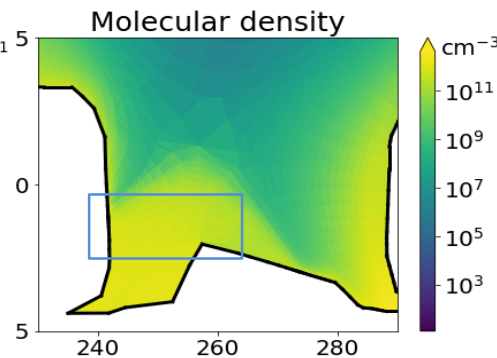
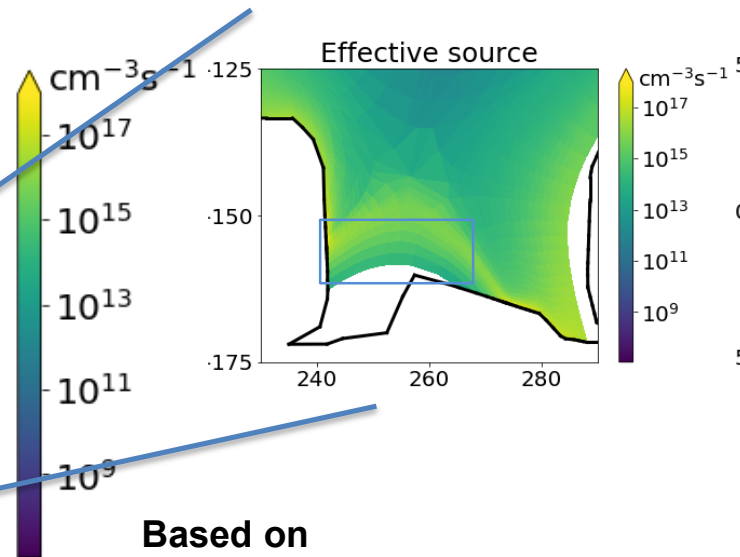
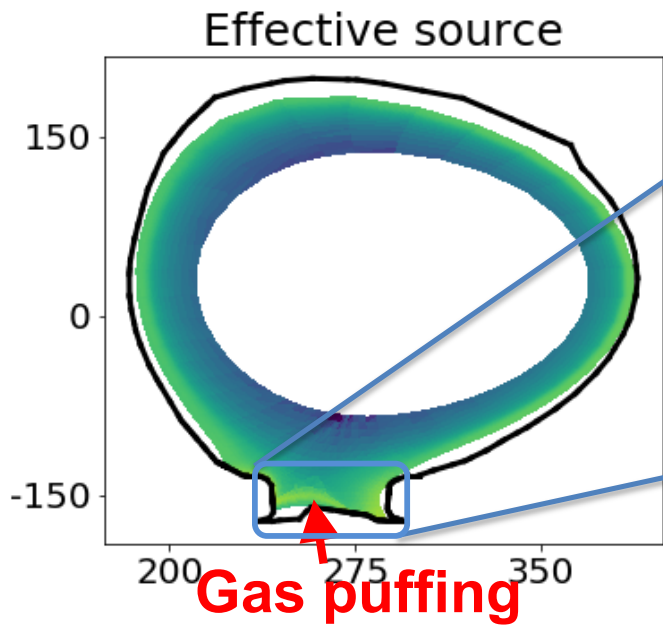


# Standalone CRM vs EIRENE: Effective source (JET)



- CRM in each simulation cell: molecule density scales linearly with the source →  
with isotope rescaling

$$\frac{\text{density}(\text{edge2D})}{\text{density}(\text{CRM})} = \frac{\text{effective source}(\text{edge2D})}{\text{reference source}(\text{CRM})}$$

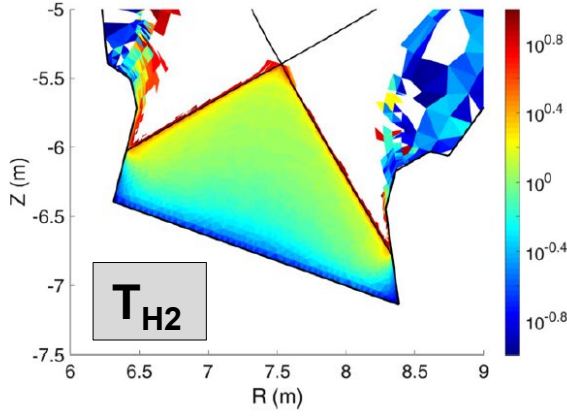
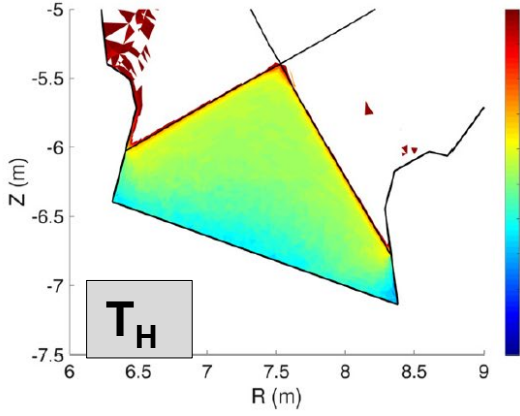
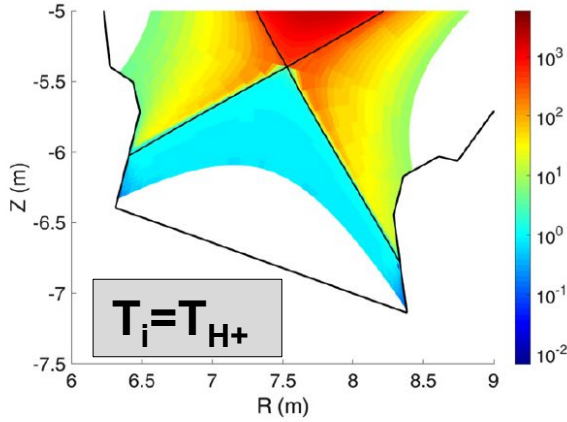
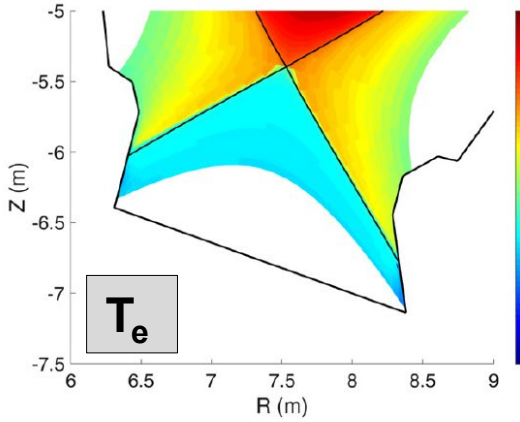


Based on  
EDGE2D-EIRENE  
simulations  
JET shot 81472

M Groth et al.  
NF 2013

- divertor region:** iny  
→ novel physical conditions (dominance of CX reactions at low temperature)

# SOLPS-ITER basic simulation case for EU-DEMO



F.Subba et al.,  
NF 61 (2021) 106013

## Below X-point:

- 1)  $T_e = T_i = 10\text{eV}$
- 2)  $T(\text{H}) = T(\text{H}_2) = 10\text{eV}$
- 3)  $N_e \sim 10^{18}\text{ m}^{-3}$
- 4)  $N(\text{H}) = 10^{20}\text{ m}^{-3}$
- 5) ...

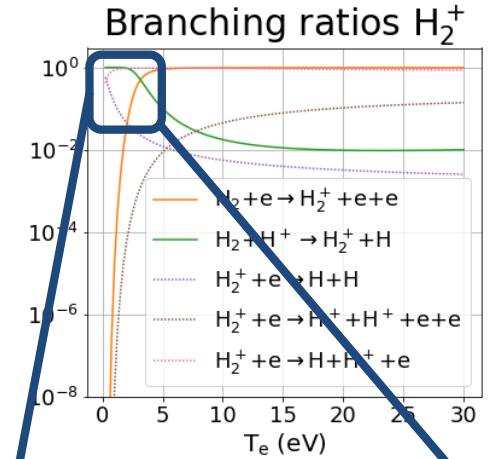
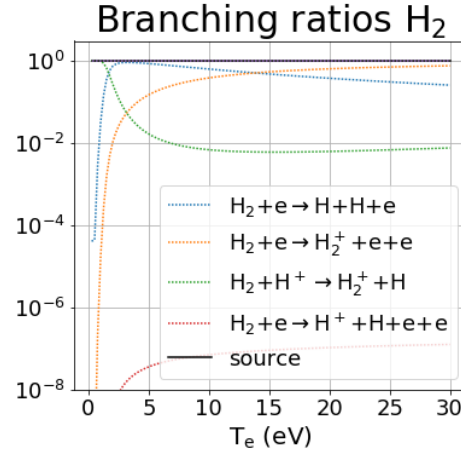
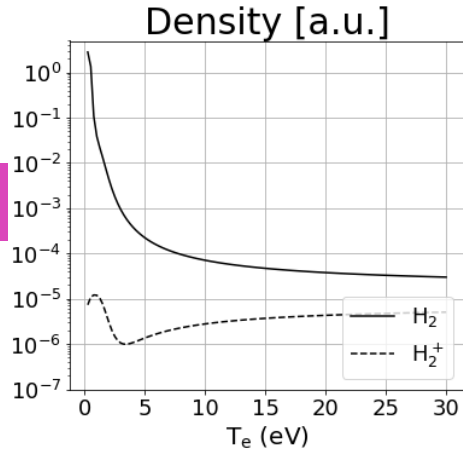
## We can do the following:

- a) *run full EIRENE: SOLPS with a "frozen" fluid side (B2.5)*
- b) *try to get some results from the standalone approach (mostly neglecting transport)*

# CRM FOR MOLECULES: leading reactions

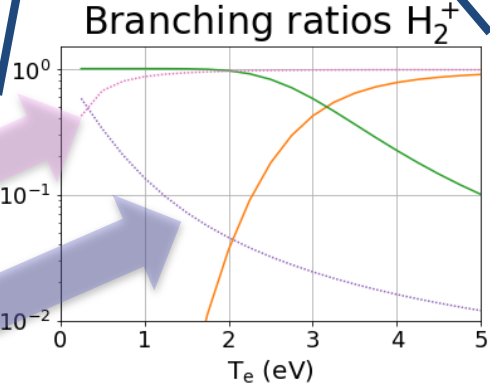
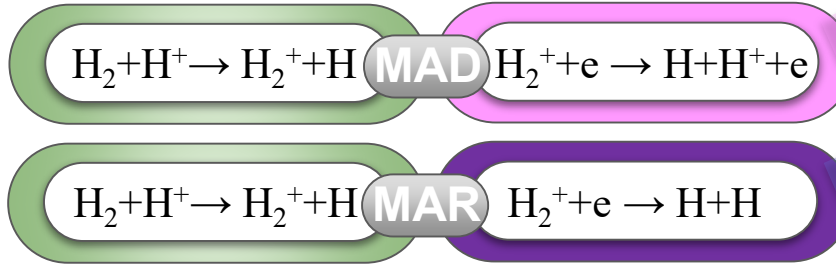


EU-DEMO



low temperature ( $T < 2\text{eV}$ ) leading reaction chains:

**MAR/MAD competition at very low temperature**





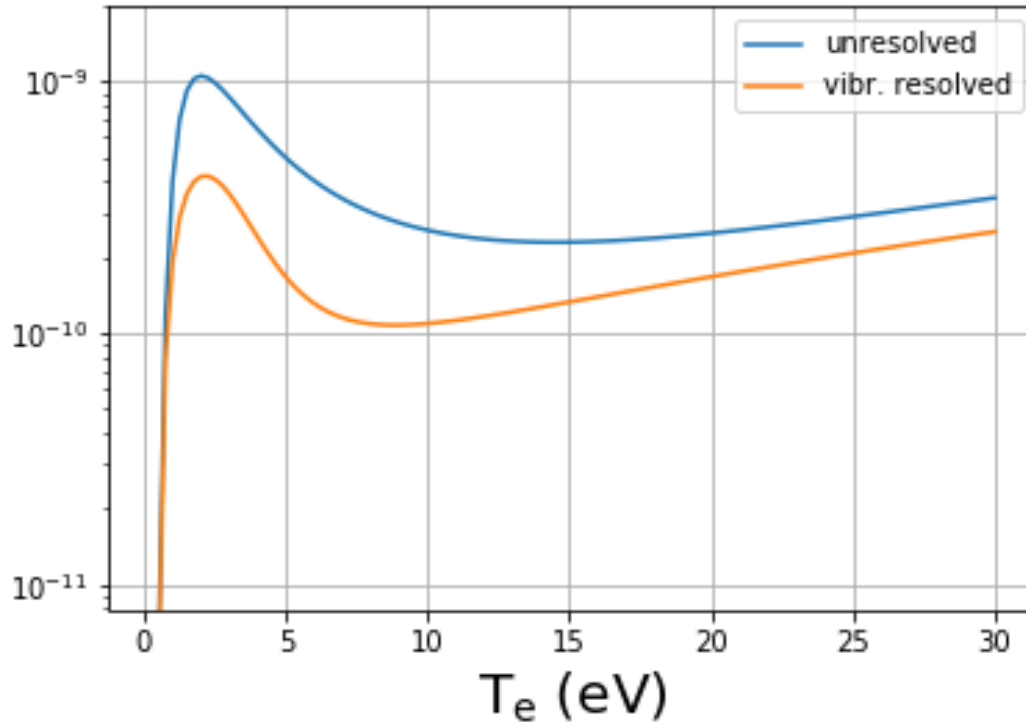
F.Cianfrani et al.,  
EPS-2022

Fully in line with the  
EIRENE and other  
CRMs:

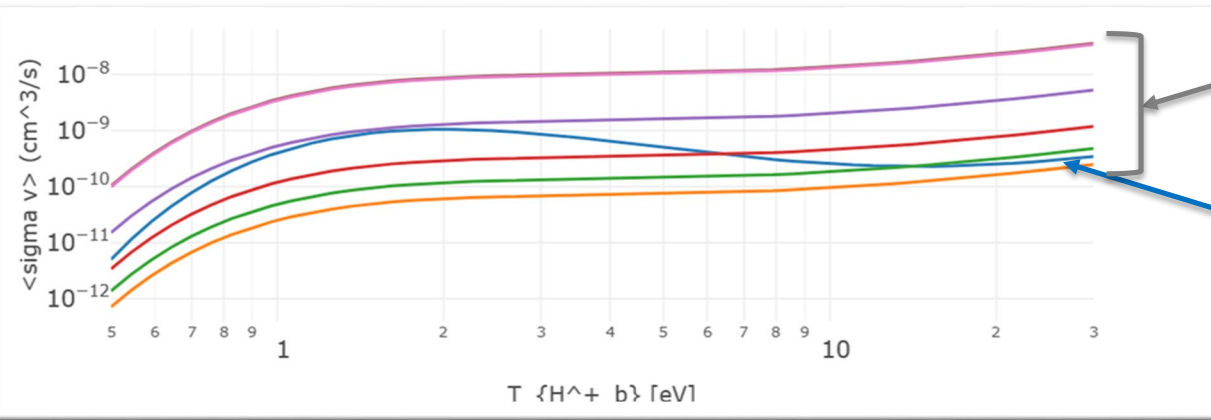
JET modelling:  
“Up to **40% reduction in effective dissociation rate** due to transport of vibrational states”

A.Holm, M.Groth, et al.,  
PET, CPP 2021

## Effective dissociation rate

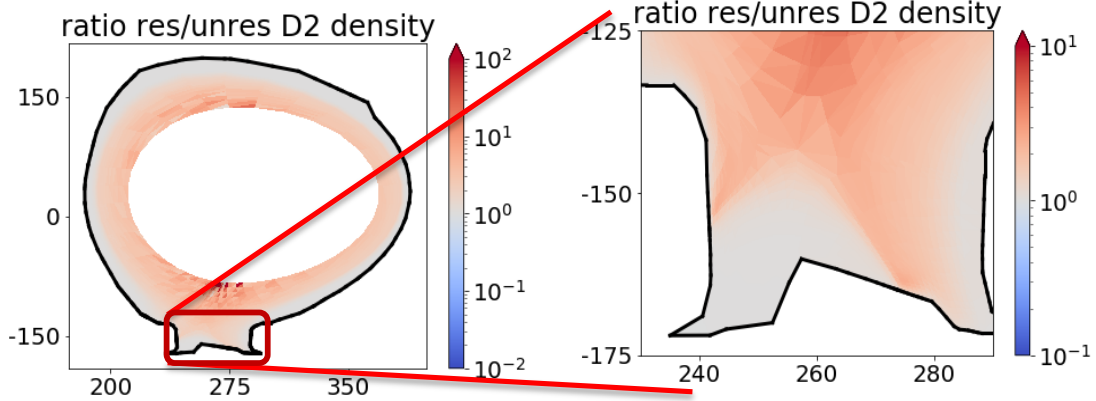


# Resolved vs. unresolved on vibrostates CRM in EIRENE



H2VIBR: resolved

AMJUEL: unresolved



**Larger molecular density (factor  $\approx 2-3$ ) along the separatrix with vibr. resolution.**

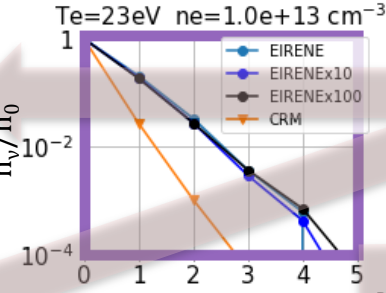
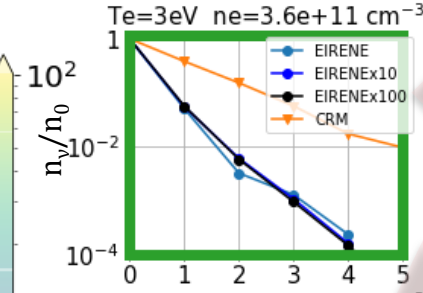
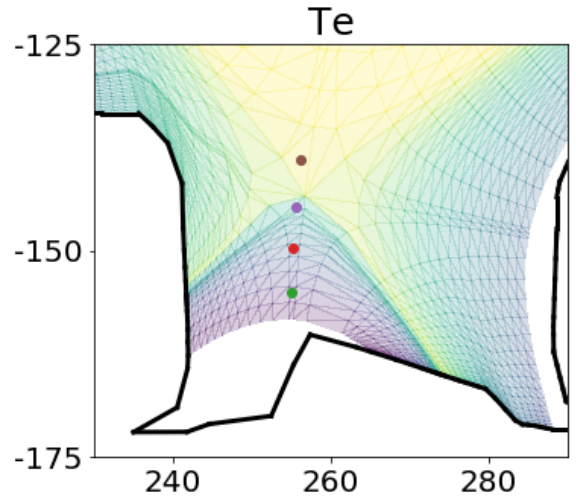


# Standalone CRM vs EIRENE - 1

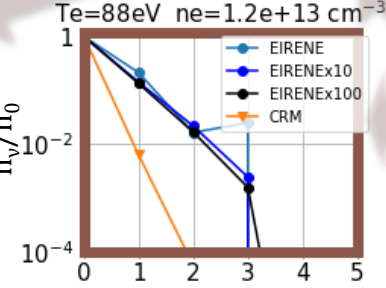
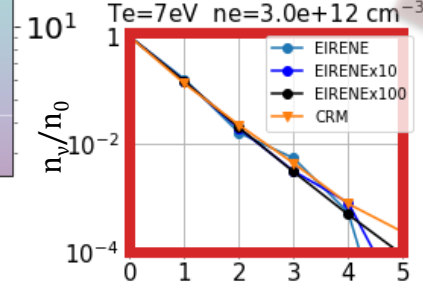


**EIRENE:** 793'300 histories  
**EIRENEx10:** 7'933'000 histories  
**EIRENEx100:** 79'330'000 histories

Simulations: SOURCE v=0



**CRM overestimates excited state population at low temperature**



**CRM underestimates excited state population at large temperature**

# Standalone CRM vs EIRENE - 2



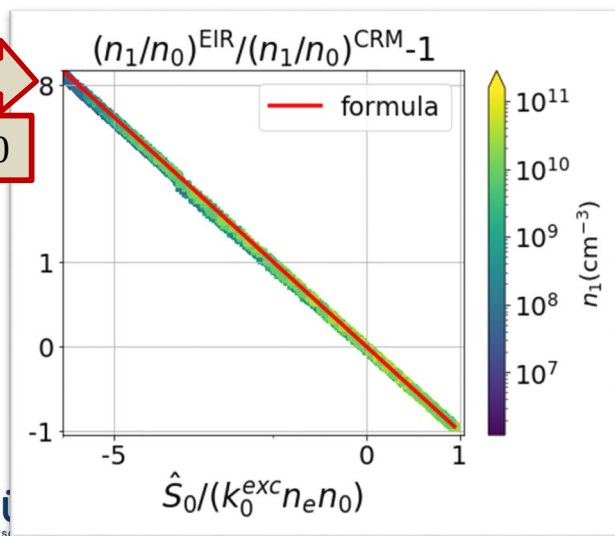
The deviations from CRM are determined by **molecular sources**:

$k_v^{exc/dex}$  : excitation/de-excitation rate

$$\left(\frac{n_{v+1}}{n_v}\right)^{EIR} = \left(\frac{n_{v+1}}{n_v}\right)^{CRM} \left(1 - \frac{\hat{S}_v}{k_v^{exc} n_e n_v}\right)$$

$$\hat{S}_v = \sum_{\bar{v}=v+1}^N S_{\bar{v}}^{EIR} \left(\frac{n_{\bar{v}}}{n_{v+1}}\right)^{CRM} \frac{k_{\bar{v}}^{dex} \dots k_{v+2}^{dex}}{k_{v+1}^{exc} \dots k_{\bar{v}-1}^{exc}}$$

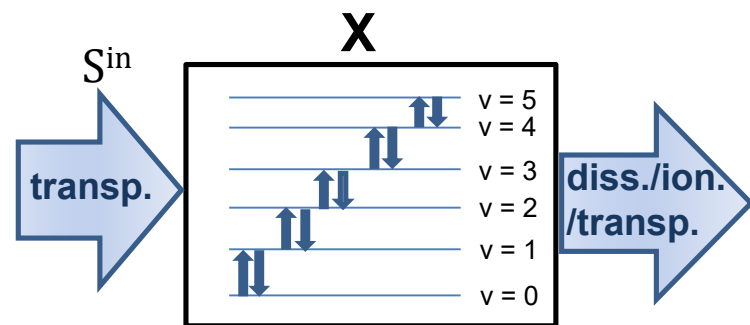
$v = 0$



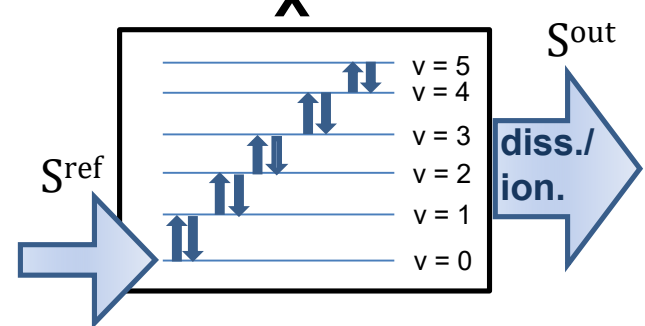
positive deviations at sinks

negative deviations at sources

EIRENE:



CRM:



# Standalone CRM vs EIRENE - 3



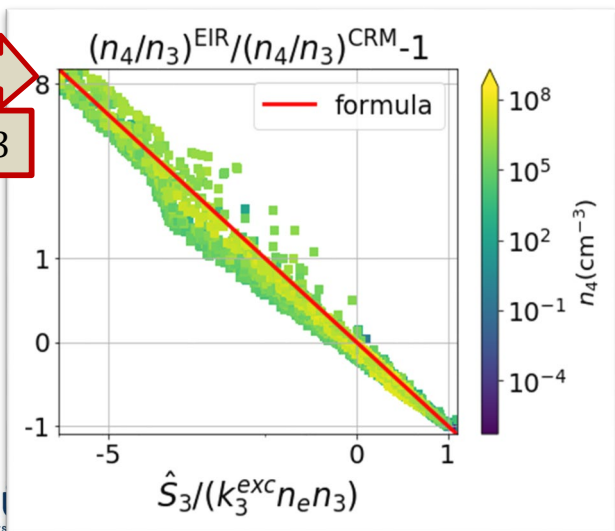
The deviations from CRM are determined by **molecular sources**:

$k_v^{exc/dex}$  : excitation/de-excitation rate

$$\left(\frac{n_{v+1}}{n_v}\right)^{EIR} = \left(\frac{n_{v+1}}{n_v}\right)^{CRM} \left(1 - \frac{\hat{S}_v}{k_v^{exc} n_e n_v}\right)$$

$$\hat{S}_v = \sum_{\bar{v}=v+1}^N S_{\bar{v}}^{EIR} \left(\frac{n_{\bar{v}}}{n_{v+1}}\right)^{CRM} \frac{k_{\bar{v}}^{dex} \dots k_{v+2}^{dex}}{k_{v+1}^{exc} \dots k_{\bar{v}-1}^{exc}}$$

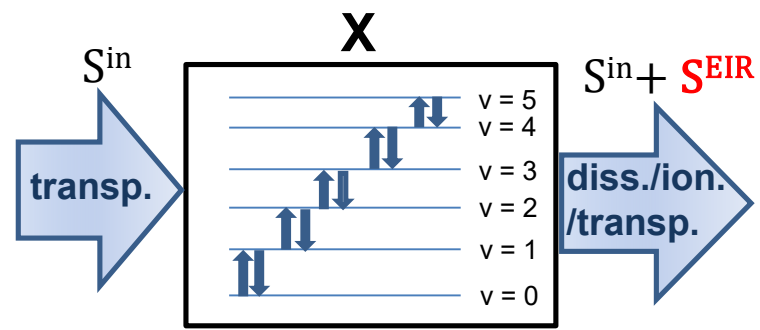
$v = 3$



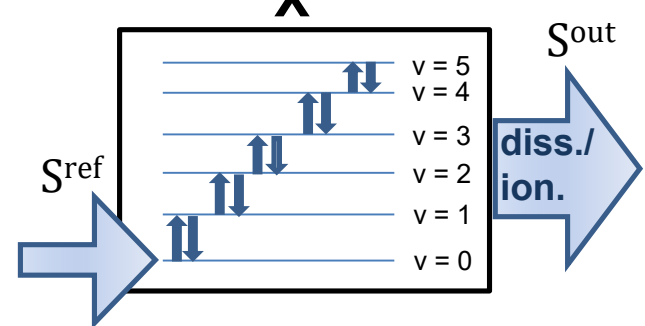
positive deviations at sinks

negative deviations at sources

EIRENE:



CRM:

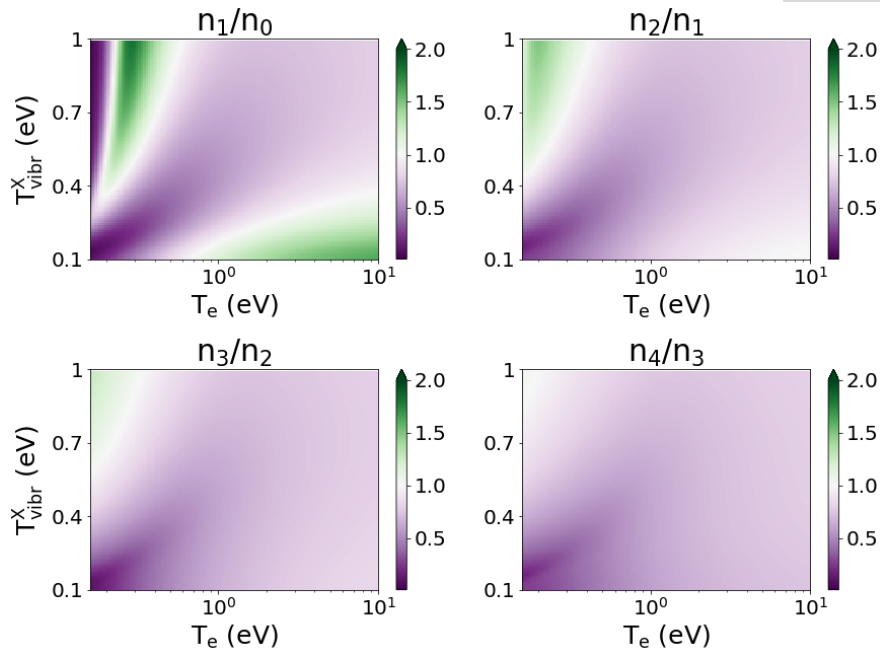


# EXCITED MOLECULAR STATES

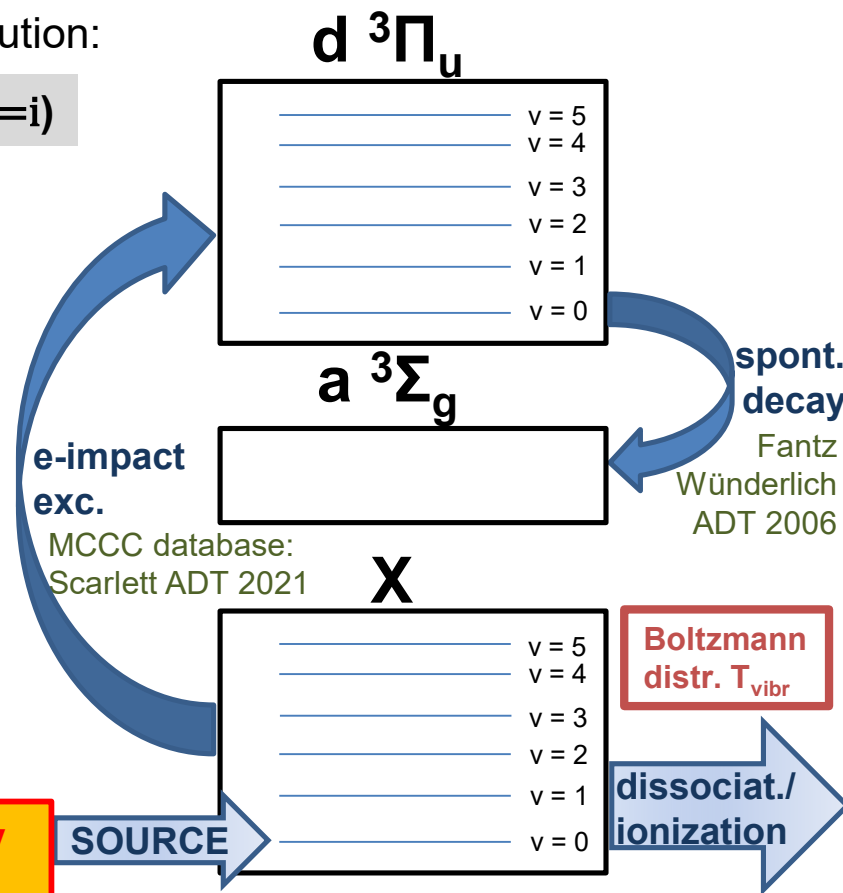


Model for excited states on top of ground state distribution:

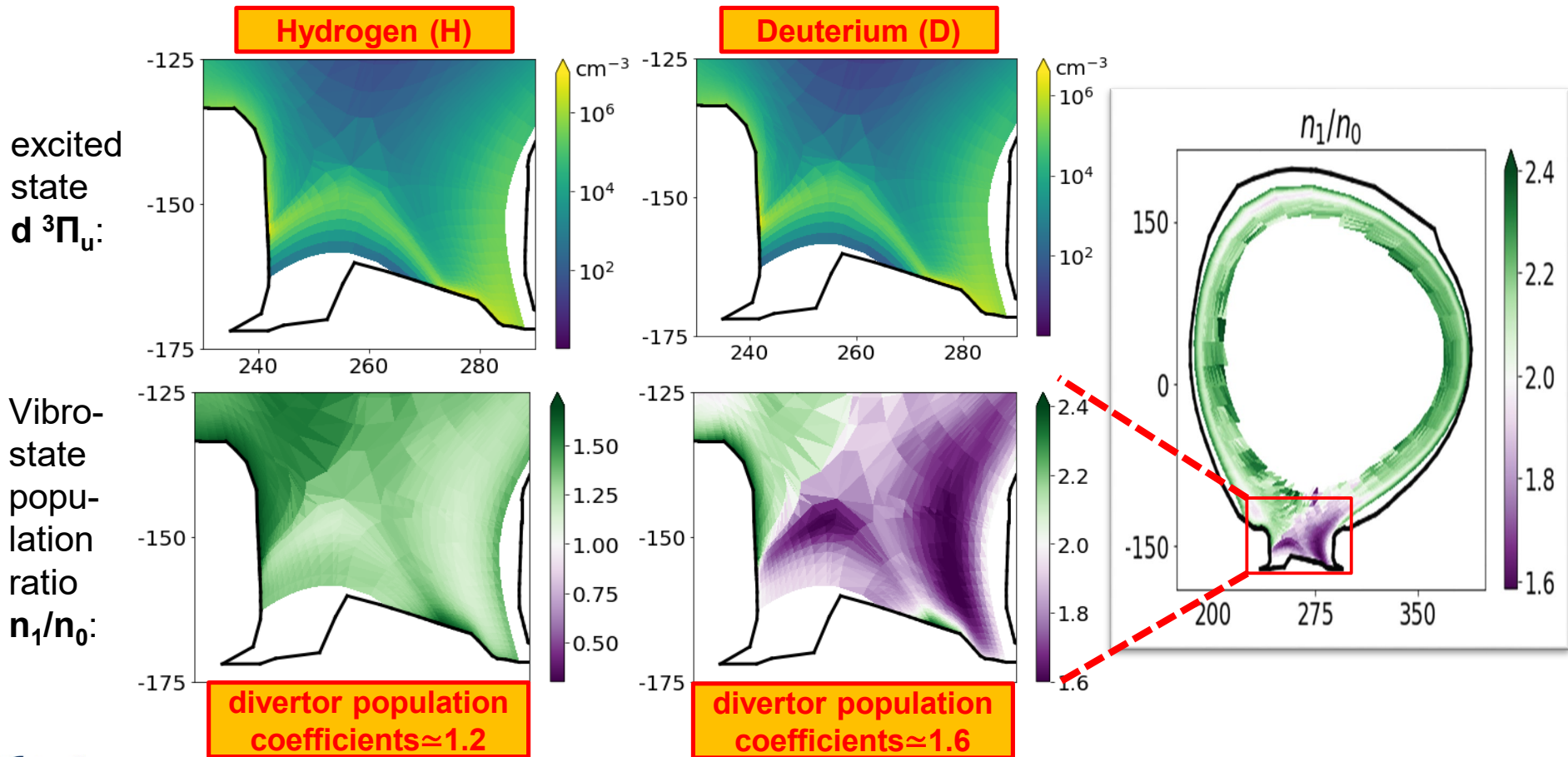
- population coeff. excited state  $d^3\Pi_u$ :  $n_i = n(d, v=i)$



**ratio of Fulcher band intensities as a proxy for ground state vibr. distribution**



# EIRENE: exited molecular states in divertor





## New (or updated) modelling tools:

→ **Ploutos**

→ **ModCR**

# Standalone CRM - Ploutos: what is inside?..



## Earlier known as “HydKin”

- 1) CRM solver (transport excluded, pure A&M side of the problem) with extensive features
  - Stationary and **non-stationary** solution (assuming velocity and plasma pars)
  - Any specie can be treated as “**Reservoir**”
  - “**Spectral analysis**” based on eigenvalue approach (which reactions are most important?)
  - **Sensitivity analysis** – vary the plasma parameters, solver settings, reactions, etc.
- 2) Plotting, solver results visualisation
  - Useful for fast **consistency checks** and trivial analysis while constructing the CRMs
  - Good for physics analysis in case one follows **established routines**
- 3) Flexible reaction tables interconnected with the solver and plotting
  - Optimised for **consistency checks** and trivial analysis while constructing the CRMs
  - **Input/output** in various formats (new: **JSON**) including directly for EIRENE
  - Critical things are highlighted, **level of information** detail **controlled** by the user.
  - Data is **grouped by the reactions** (even if from various sources).
  - Standard or custom **initial configurations**

Power of  
web-based  
Interface!

# PLOUTOS can be used to



[www.eirene.de](http://www.eirene.de)



*Both statues are at Glyptothek, Munich.*

**EIRENE**

- to import/export data (JSON, tabular, etc.)
- to produce input data for EIRENE and for other codes with CRMs
  - ➔ *load/improve/save the developed configuration (selected reactions and parameters) including starting from the standard pre-sets*
- to check data for consistency and abnormal features
- do sensitivity studies:
  - ➔ *understand A&M side of the problem and identify the most significant processes (among the selected ones)*



# PLOUTOS – (H<sub>2</sub> 2022/23 case)



Collision with e						
not included in solver	selected data unselect all	plot unselect all	Number	reaction	range	reference
<input checked="" type="radio"/>	<input type="radio"/>	<input checked="" type="checkbox"/>	2.1.1	e + H(1s) → H(2p) + e	E_min: 1.08e+01 - E_max: 2.00e+04	JanevEtAl(1987)
<input type="radio"/>	<input type="radio"/>	<input checked="" type="checkbox"/>		e + H(1s) → H(2p) + e	T_min: 1.26e+00 - T_max: 2.00e+04	JanevEtAl(1987)
<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="checkbox"/>	2.1.2	e + H(1s) → H(2s) + e	E_min: 1.08e+01 - E_max: 2.00e+04	JanevEtAl(1987)
<input type="radio"/>	<input type="radio"/>	<input checked="" type="checkbox"/>		e + H(1s) → H(2s) + e	T_min: 1.26e+00 - T_max: 2.00e+04	JanevEtAl(1987)
<input type="radio"/>	<input type="radio"/>	<input type="checkbox"/> n=1 -> n'=2 <input type="checkbox"/> n=2 -> n'=1		e + H(n) → H(n') + e	T_min: 1.00e-01 - T_max: 1.00e+03 n_min: 1.00e+08 - n_max: 1.00e+16	SawadaFujimoto(199

Visibility columns:

- reaction: switch to off
- range: switch to off
- reference: switch to off
- data type: switch to off
- Peculiar properties: switch to on
- generation: switch to on
- data origin: switch to on
- File/chapter: switch to off

**Buttons to adjust table content**

show only selected reactions (Groups)

show only selected reactions (rows)

# New CRM Solver for EIRENE concepted



- ❑ This CRM is aimed to precompute rate coefficients accounting for **all parametric dependences** ( $n_e$ ,  $T_e$ , but also  $T_i$ , ...) in contrast with currently used polynomial fits (AMJUEL, ...) + add a number of levels/processes not accounted for at this time
- ❑ The **internal states** (e.g. rovibrational states in molecular species) are to be tracked with a flexible a flexible control over this resolution (as separate specie or variable).
- ❑ The **nonstationary solution** for balance equations should be the default one (with the stationary only as a useful option).
- ❑ The solver should be **modular**, thus **usable standalone** or even in **various codes**.
- ❑ The **improved A&M data input** should be readable and structured (for starters JSON, potentially also HDF5). It should be pre-processed mostly automatic and easily exchanged with other codes and tools. We need **tools for visualisation and testing**.
  - The only way to meet the exploding amounts of data from RMPS and CCC for molecules (with resolution by rovibrational states)
  - IAEA GNAMPP assists, but also reveals the challenges

$$T_i \neq T_e, \\ \text{etc.}$$

$$\frac{dN_i}{dt} \neq 0$$

*Why not  
also ERO?..*

***Not only performance and reliability to be improved, but additional physics can be provided!***

# New code (also library for EIRENE): “ModCR”



- Should provide **BOTH** standalone and build-in CRM for EIRENE
  - ❑ Therefore written in modern Fortran
  - ❑ Linked with Plutos (uses the same JSON files for I/O)
    - (AMJuel, HydHel, H2Vibr, etc are already inside, moreover under flexible web-powered control)*
  - ❑ Non-stationary approach utilising up-to-date Sundails solver (CVODE) for stiff ODE systems
    - (algebraic stationary option also available for control)*
- The whole specie-state and reaction basis is inside, however the code should form the list of “active states”
  - ❑ Determined by user in JSON parameter file (for a few simulations)
  - ❑ Provide different resolution on states being tracked
  - ❑ Provide **flexible border** for tracking of MC species and internal state populations as variables



**This**  
is unique  
and justifies  
creation of  
the new  
code



## • SUNDIALS: SUite of Nonlinear and Differential/ALgebraic Equation Solvers

Developed at LLNL - awarded the 2023 SIAM/ACM Prize in Computational Science and Engineering

 repository: <https://github.com/LLNL/sundials>



Package	Purpose
<b>CVODE</b>	solves initial value problems for ordinary differential equation (ODE) systems.
<b>CVODES</b>	solves ODE systems and includes sensitivity analysis capabilities (forward and adjoint).
<b>ARKODE</b>	solves initial value ODE problems with additive Runge-Kutta methods, including support for IMEX methods.
<b>IDA</b>	solves initial value problems for differential-algebraic equation (DAE) systems.
<b>IDAS</b>	solves DAE systems and includes sensitivity analysis capabilities (forward and adjoint).
<b>KINSOL</b>	solves nonlinear algebraic systems.

**Modern, powerful and flexible – still it is available in FORTRAN (so suits to EIRENE)**



- 1) **CRMs are essential and potentially common part of the plasma MC transport codes**
- 2) **Transport and A&M sides of MC transport codes are NOT separable, however using the standalone approach is often extremely useful**
  - Gives insight into the A&M physics
  - Allows more control, looking into details and straightforward answers
  - Particular valuable if standalone and external CRM would be just the same
  - Isotope (D vs. H) effect demonstrates the significance of vibro-resolved MCCC data
- 3) **New CRM (ModCR) is concepted and under development**
  - PLOUTOS will keep and even strengthen its role as data pre-processor
  - It is aimed to provide flexible “border” between particles treated as MC species or variables characterising the internal state
- 4) **Opacity and other effects should be available in ModCR, but it is too early to talk about that at the current stage.**

Further details at [www.eirene.de](http://www.eirene.de)



# Thanks for the attention!

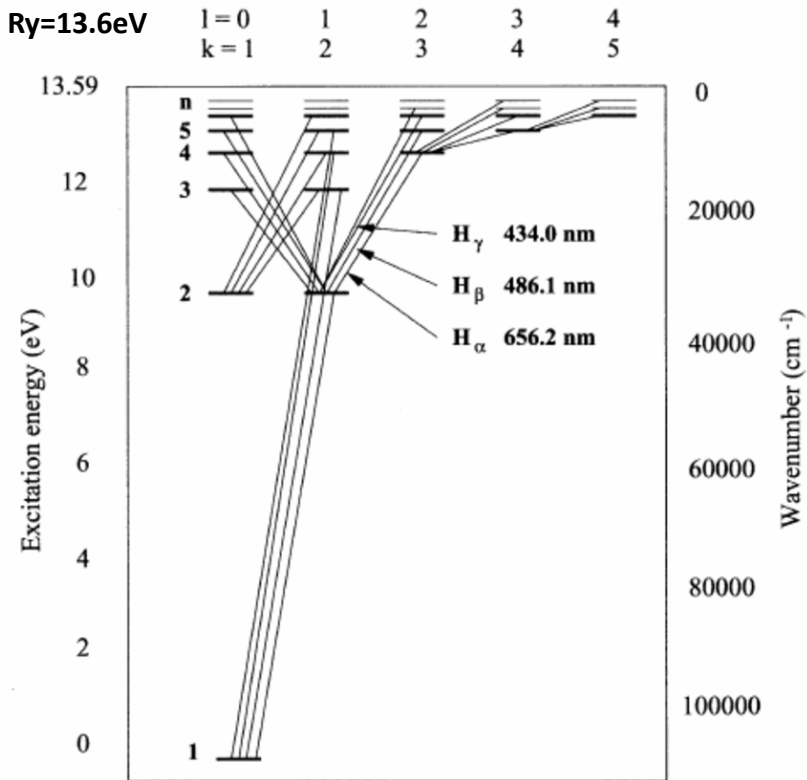
## Special thanks to

- 1) ADAS (M.O'Mullane et al.) for constant support
- 2) YACORA developers (U.Fanz, D.Wunderlich, et al.) for consultations
- 3) MCCC developers (D.V.Fursa et al.) for providing the database

# Collisional-Radiative Models (CRM)



Grotrian diagram for atomic H (D, T)



## Balance equations:

$$\frac{dN_i}{dt} = \sum_{j \neq i} A_{ji} N_j + n_e \cdot (EXCIT + IZ + CX + REC)$$

$$EXCIT = \sum_{j \neq i} \langle v \sigma_{ji} \rangle N_j$$

$$IZ = \sum_m \langle v \sigma_{mi} \rangle N_m^- + \sum_z \langle v \sigma_{zi} \rangle N_z^{2-} + \dots$$

$$REC = \sum_k \langle v \sigma_{ki} \rangle N_k^+ + \sum_l \langle v \sigma_{li} \rangle N_l^{2+} + \dots$$

...

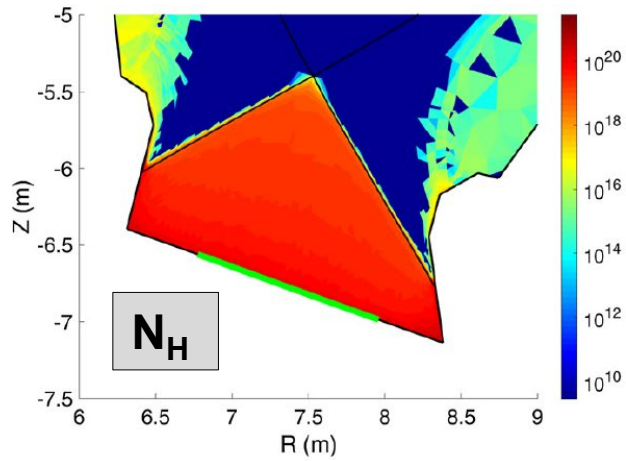
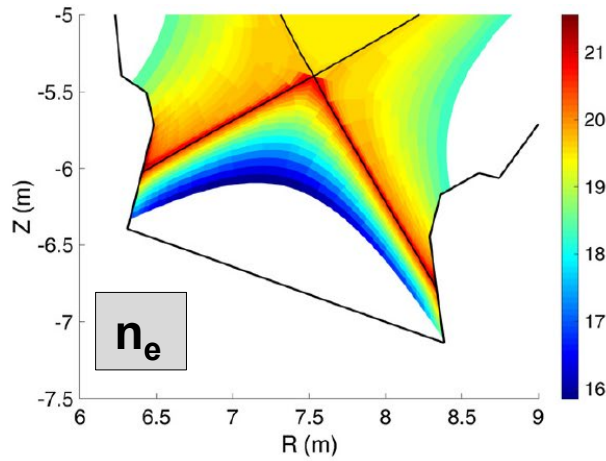
*j, k, l, m, z, ... states can be fine-superfine resolved or, opposite, bundled into few quasi-metastables (MS)*

CRM = list of states + transition data

Often used:

$\langle v \sigma_{ji} \rangle (T_e, n_e)$  - effective Maxwellian averaged rates

# SOLPS-ITER basic simulation case for EU-DEMO



F.Subba et al.,  
NF 61 (2021) 106013

- Below X-point: ★
- 1)  $T_e = T_i = 10\text{eV}$
  - 2)  $T(\text{H}) = T(\text{H}_2) = 10\text{eV}$
  - 3)  $N_e \sim 10\text{e}18\text{ m}^{-3}$
  - 4)  $N(\text{H}) = 10\text{e}20\text{ m}^{-3}$

- ❑ These are semi-detached conditions (1 point)
  - ➔ Attached vs. detached + 2-3 semidetached cases would be wishful to have!
  - ➔ We use EIRENE or standalone CRM to identify spectral features useful for the degree of detachment control for this plasma conditions
- ❑ We can run (and mean to in future!) full EIRENE: SOLPS with a “frozen” fluid side (B2.5)
  - ➔ This option is not mature enough for meaningful physics results
  - ➔ We use standalone CRM and vary constant plasma conditions around ★





- **SUNDIALS**: SUite of Nonlinear and Differential/ALgebraic Equation Solvers

written in C++: **Modern Fortran interface** (modules)

$$\dot{y} = f(t, y), \quad y(t_0) = y_0$$

□ **CVODE**: solution of IVP  
variable-step multistep methods: 
$$\sum_{i=0}^{K_1} \alpha_{n,i} y^{n-i} + h_n \sum_{i=0}^{K_2} \beta_{n,i} \dot{y}^{n-i} = 0$$

- ADAMS-MOULTON FORMULAS
- BACKWARDS DIFFERENTIATION FORMULAS → stiff problems

**Nonlinear solve:**

- default: Newton iteration based on linear solver
- user-defined

**Linear solver families:**

- direct for dense, banded, or sparse matrices
- spils: scaled preconditioned iterative (Krylov) linear solvers.



# YACORA: inclusion molecular source term



- Stationary run vibrationally resolved:

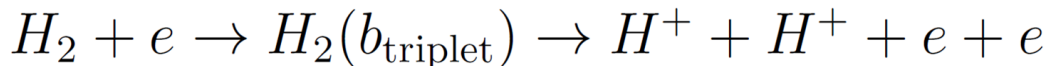
Particle species

$$p, H_2^+, Z$$

$$H_2(v=0), \dots, H_2(v=14)$$

Reactions

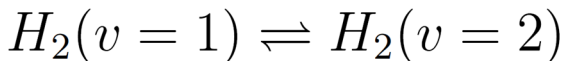
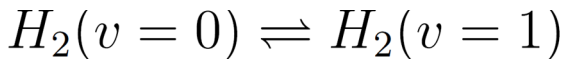
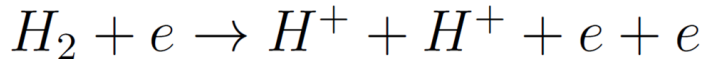
electron impact



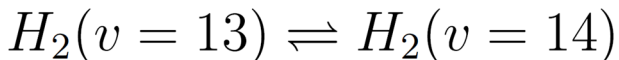
proton impact



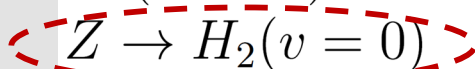
electron impact



.....



H<sub>2</sub> source term



YACORA + Source term  
adjustable to the Edge2D-EIRENE

F.Cianfrani,  
EPS-2022

H<sub>2</sub> density (m<sup>-3</sup>)

