



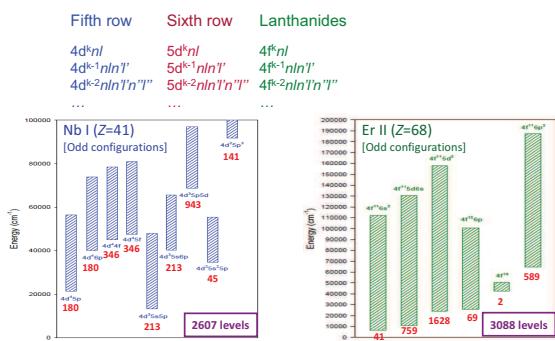
Atomic structure and radiative data calculations for heavy elements of interest in fusion plasma research

Theoretical challenges and recent advances

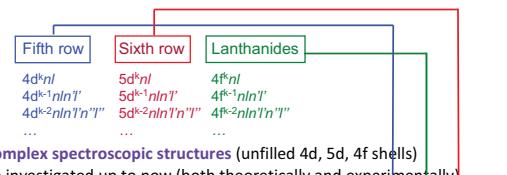
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Introduction – Motivations



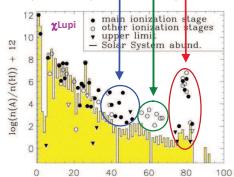
Introduction – Motivations



Interest in many other fields :

- Astrophysics (CP stars, nucleosynthesis, ...)
- Solid state physics (doped crystals, ...)
- New light sources (lasers, lamps, ...)
- Plasma physics (fusion, ITER, ...)

Progress in computers & laser spectroscopy
→ New systematic investigations possible



Theoretical approach

Relativistic Hartree-Fock method (HFR)

(R.D. Cowan, *The Theory of Atomic Structure and Spectra*, Univ. California Press, Berkeley, 1981)

Based on the Schrödinger equation (atom with N electrons)

$$H\Psi = E\Psi \quad \text{with} \quad H = \sum_{i=1}^N \left[-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{Ze^2}{4\pi\epsilon_0 r_i} + \sum_{j<i} \frac{e^2}{4\pi\epsilon_0 r_{ij}} \right]$$

Central field approximation

$$\varphi_{nlm_s}(\mathbf{q}) = \frac{1}{r} P_{nl}(r) Y_{lm_l}(\theta, \phi) \chi_{1/2, m_s}(\sigma)$$

Slater determinant

$$\Psi(\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_1(\mathbf{q}_1) & \varphi_2(\mathbf{q}_1) & \dots & \varphi_N(\mathbf{q}_1) \\ \varphi_1(\mathbf{q}_2) & \varphi_2(\mathbf{q}_2) & \dots & \varphi_N(\mathbf{q}_2) \\ \dots & \dots & \dots & \dots \\ \varphi_1(\mathbf{q}_N) & \varphi_2(\mathbf{q}_N) & \dots & \varphi_N(\mathbf{q}_N) \end{vmatrix}$$

Introduction – Motivations

The Periodic Table of the Elements

1	2	3	4	5	6	7	8	9	10
H	He	Li	Be	B	C	N	O	F	Ne
Hydrogen	Helium	Lithium	Boron	Boron	Carbon	Nitrogen	Oxygen	Fluorine	Neon
1	2	3	4	5	6	7	8	9	10
Li	Be	Be	Be	B	C	N	O	F	Ne
3	4	5	6	7	8	9	10	11	12
Na	Mg	Mg	Mg	Mg	Mg	Mg	Mg	Mg	Mg
11	12	13	14	15	16	17	18	19	20
K	Ca	Ca	Sc	Ti	V	Cr	Mn	Fe	Co
19	20	21	21	22	23	24	25	26	27
Rb	Sr	Sr	Zr	Zr	Y	Cr	Mn	Fe	Co
37	38	39	40	41	42	43	44	45	46
Cs	Fr	Sr	Zr	Zr	Y	Cr	Mn	Fe	Co
55	87	88	89	105	106	107	108	109	110
Ba	Ra	Ra	Ac	Rf	Db	Sg	Bh	Mt	Tl
87	88	89	105	106	107	108	109	110	111
Fr	Ra	Ra	Ac	Rf	Db	Sg	Bh	Mt	Tl
121	121	121	121	121	121	121	121	121	121

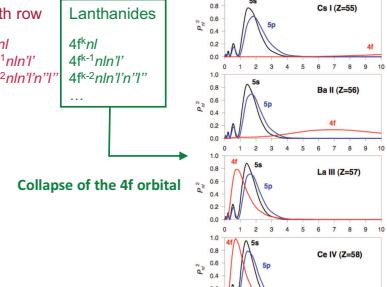
Introduction – Motivations

Fifth row Sixth row Lanthanides

4d^knl
4d^{k-1}nl'n'l'
4d^{k-2}nl'n'l'n'l'
...

5d^knl
5d^{k-1}nl'n'l'
5d^{k-2}nl'n'l'n'l'

4f^knl
4f^{k-1}nl'n'l'
4f^{k-2}nl'n'l'n'l'



Determination of atomic data

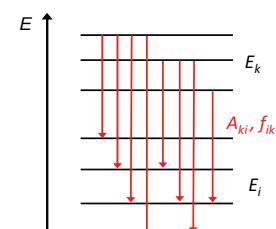
Theoretically and experimentally...

Atomic structure parameters

- Energy levels, E_k, E_i
- Transition energies, ΔE_{ki}
- Wavelengths, λ_{ki}

Radiative parameters

- Transition probabilities, A_{ki}
- Oscillator strengths, f_{ik}
- Radiative lifetimes, $\tau_k = 1/\sum_i A_{ki}$



Theoretical approach

Relativistic Hartree-Fock method (HFR)

(R.D. Cowan, *The Theory of Atomic Structure and Spectra*, Univ. California Press, Berkeley, 1981)

Hartree-Fock equations

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dr_i^2} + \frac{\hbar^2 l_i(l_i+1)}{2mr_i^2} - \frac{Ze^2}{4\pi\epsilon_0 r_i} \right] P_i(r_i) + \left[\sum_{j \neq i} \int |P_j(r_j)|^2 \frac{e^2}{4\pi\epsilon_0 r_{ij}} dr_j \right] P_i(r_i) - \left[\sum_{j \neq i} \delta_{m_j, m_i} \delta_{m_j, m_i} P_j^*(r_j) P_j(r_i) \frac{e^2}{4\pi\epsilon_0 r_{ij}} dr_j \right] P_j(r_i) = E_i P_i(r_i)$$

Resolution of Hartree-Fock equations

Iterative method → self-consistent field method

Theoretical approach

Relativistic Hartree-Fock method (HFR)

(R.D. Cowan, *The Theory of Atomic Structure and Spectra*, Univ. California Press, Berkeley, 1981)

Multiconfiguration approach

$$\Psi^k = \sum_b a_b^k \Psi_b$$

Relativistic effects

Included perturbationally (spin-orbit, mass-velocity, Darwin term)

Good agreement with fully relativistic methods

Ab initio or semi-empirical approach

Experimental energy levels can be used to optimize the radial parameters

Theoretical approach

Relativistic Hartree-Fock method (HFR)

(R.D. Cowan, *The Theory of Atomic Structure and Spectra*, Univ. California Press, Berkeley, 1981)

Multiconfiguration approach

$$\Psi^k = \sum_b a_b^k \Psi_b$$

Example : Lu III (Lu^{2+})
(Ground config. : $[\text{Xe}]4f^{14}5s^25p^{6s}$)

Intravalance correlation
(up to $n=6, l=3$)

Even parity Odd parity

$5s^25p^6s$ $5s^25p^6p$
 $5s^25p^5d$ $5s^25p^5f$

$5s^25p^6d$ $5s^25p^6s^2$
 $5s^25p^5f6s$ $5s^25p^6p^2$

$5s^25p^5f6d$ $5s^25p^6d^2$
 $5s^25p^6s6p$ $5s^25p^6f^2$

$5s^25p^6s6f$ $5s^25p^6d6s$
 $5s^25p^6p6d$ $5s^25p^6f6p$

$5s^25p^6d6f$ $5s^25p^6f6p$
 $5s^25p^6s6d$ $5s^25p^6d6d$

$5s^25p^6p6f$ $5s^25p^6d6f$

5 levels

6 levels

Core-valence correlation (with 5s, 5p)

Even parity Odd parity

$5s^25p^6s5d5f$ $5s^25p^5s5d^2$
 $5s^25p^5d6p$ $5s^25p^5f^2$
 $5s^25p^6s^2$ $5s^25p^6d^2$
 $5s^25p^5f6s$ $5s^25p^6p^2$
 $5s^25p^5f6d$ $5s^25p^6d^2$
 $5s^25p^6s6p$ $5s^25p^6f^2$
 $5s^25p^6s6f$ $5s^25p^6d6s$
 $5s^25p^6p6d$ $5s^25p^6f6p$

885 levels

904 levels

Core-polarization effects (HFR + CPOL)

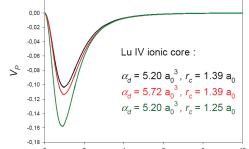
(see e.g. Quinet et al., M.N.R.A.S. **307**, 934, 1999; Quinet et al., J. Alloys Compd **344**, 255, 2002)

Intravalance correlation considered within a configuration interaction scheme

Core-valence correlation represented by a **core-polarization model potential** depending on two parameters (dipole polarizability α_d and cut-off radius r_c)

$$V_{P1} = -\frac{1}{2} \alpha_d \sum_{i=1}^n \frac{r_i^2}{(r_i^2 + r_c^2)^3}$$

$$V_{P2} = -\alpha_d \sum_{i>j} \frac{\vec{r}_i \cdot \vec{r}_j}{[(r_i^2 + r_c^2)(r_j^2 + r_c^2)]^{3/2}}$$



Correction to the dipole radial integral

$$\int_0^\infty P_{nl}(r) r P_{nl'}(r) dr \quad \text{replaced by} \quad \int_0^\infty P_{nl}(r) r \left[1 - \frac{\alpha_d}{(r^2 + r_c^2)^{3/2}} \right] P_{nl'}(r) dr$$

Semi-empirical optimization

(R.D. Cowan, *The Theory of Atomic Structure and Spectra*, Univ. California Press, Berkeley, 1981)

Radial parameters (average energies, electrostatic integrals, spin-orbit parameters) adjusted in order to minimize the discrepancies between computed Hamiltonian eigenvalues and experimental energy levels

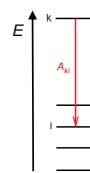
→ Optimization of the transition energies and wavefunctions

→ Optimization of the radiative decay rates

$$A_{ki} = \frac{64\pi^4 e^4 a_0^2 (\Delta E_{ki})^3}{3h} \left| \langle \gamma_i J_i \| P^{(1)} \| \gamma_k J_k \rangle \right|^2$$

$$= 2.0261 \times 10^{-6} \frac{(\Delta E_{ki})^3}{2J_k + 1} \left| \langle \gamma_i J_i \| P^{(1)} \| \gamma_k J_k \rangle \right|^2$$

→ A good experimental knowledge of the level structure for the atom (or ion) considered is needed



Experimental measurements

Time-resolved laser-induced fluorescence (TR-LIF)

(see e.g. Bergström et al., Z. Phys. D **8**, 17, 1988; Xu et al., Phys. Rev. A **70**, 042508, 2004)

Accurate measurements of radiative lifetimes (within a few %)

Lifetime range from 1 ns to 300 ns

Selective excitation (no cascading problems)

Many levels accessible (using different laser dyes)

Different ionization degrees accessible in the laser-produced plasma (neutral, singly-, doubly- and trebly-ionized atoms)

A specific example (Lu III)

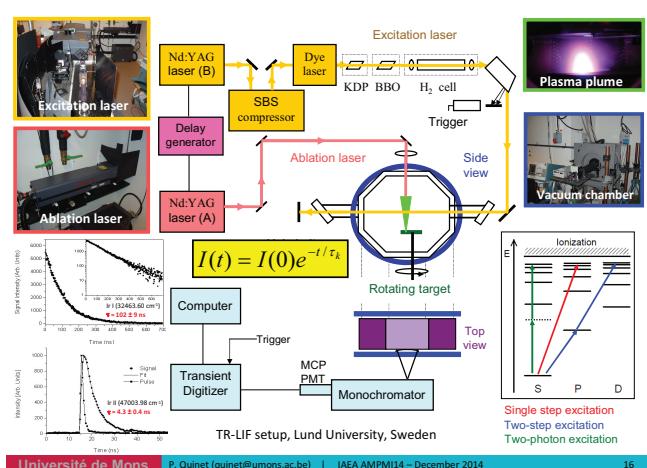
(Biémont et al., J. Phys. B **32**, 3409, 1999)

Radiative lifetimes for $6p^2P_{1/2}$ (38401 cm^{-1}) and $6p^2P_{3/2}$ (44705 cm^{-1})

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 / n!$$

Lu³⁺ ionic core with 68 electrons :
 $\alpha_d = 5.20 a_0^{-3}$ (Fraga et al., 1976)
 $r_c = <\!r>_{5p} = 1.39 a_0$

Method	$2P_{1/2}$	$2P_{3/2}$
HF	1.49	1.00
HFR	1.57	1.06
HFR+POL	2.03	1.38
HFR+POL+PEN	2.23	1.47
Experiment	2.20 ± 0.20	1.55 ± 0.20



The specific case of tungsten (W I, W II, W III)

Transition rates in W I, W II and W III

W I : 508 experimentally known energy levels within the

5d⁴6s², 5d⁵6s, 5d⁴6s7s, 5d⁵6p, 5d⁴6s6p and 5d³6s²6p configurations
(Kramida & Shirai, J. Phys. Chem. Ref. Data **35**, 423, 2006; Wyart, J. Phys. B **43**, 074018, 2010)

W II : 263 experimentally known energy levels within the

5d⁵, 5d⁴6s, 5d³6s², 5d⁴6p, 5d³6s6p and 5d²6s²6p configurations
(Kramida & Shirai, J. Phys. Chem. Ref. Data **35**, 423, 2006)

W III : 235 experimentally known energy levels within the

5d⁴, 5d³6s, 5d²6s², 5d³6p and 5d²6s6p configurations
(Iglesias et al., J. Res. Natl Inst. Stand. Tech. **94**, 221, 1989)

Transition probabilities and oscillator strengths calculated for :

2525 W I lines in the range 223 – 1010 nm (Quinet et al., J. Phys. B **44**, 145005, 2011)

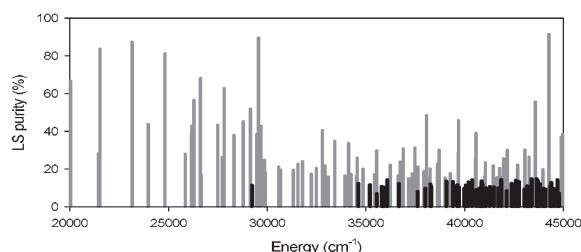
6086 W II lines in the range 143 – 990 nm (Nilsson et al., Eur. Phys. J. D **49**, 13, 2008)

4822 W III lines in the range 83 – 1494 nm (Palmeri et al., Phys. Scr. **78**, 015304, 2008)

The specific case of tungsten (W I, W II, W III)

Transition rates in W I

Wavefunction purities (in LS coupling) for W I odd-levels below 45000 cm⁻¹
(bold lines represent purities smaller than 15%)



The specific case of tungsten (W I, W II, W III)

Critical evaluation of available data



The specific case of tungsten (W IV, W V, W VI)

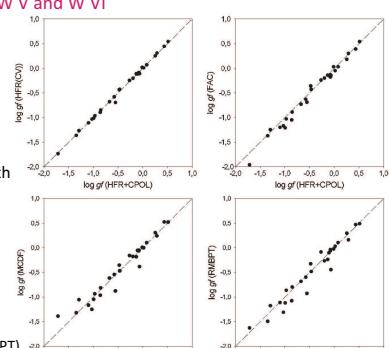
Transition rates in W IV, W V and W VI

No experimental radiative data to compare with !

▪ Use of different and independent theoretical methods

▪ HFR + CPOL compared with

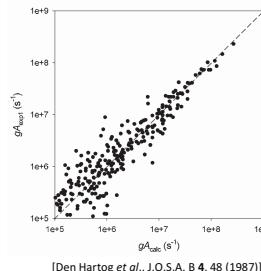
- HFR including core-valence (HFR(CV))
- Flexible Atomic Code (FAC)
- Multiconfiguration Dirac-Fock (MCDF)
- Relativistic Many-Body Perturbation Theory (RMBPT)



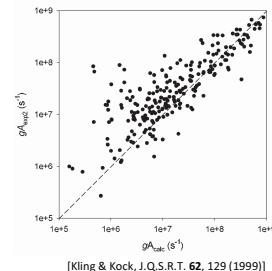
The specific case of tungsten (W I, W II, W III)

Transition rates in W I

Comparison between theoretical transition probabilities and available experimental results



[Den Hartog et al., J. O.S.A. B **4**, 48 (1987)]



[Kling & Kock, J.Q.S.R.T. **62**, 129 (1999)]

The specific case of tungsten (W I, W II, W III)

Transition rates in W II and W III

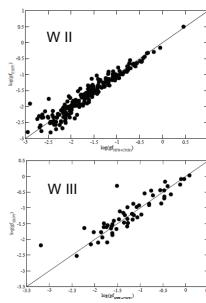
Comparison between theoretical and experimental lifetimes obtained in our work

E (cm ⁻¹)	J	Theory	Experiment
47179.941	3/2	6.8	5.8 ± 0.3
48284.498	5/2	7.6	5.9 ± 0.3
48181.034	9/2	20.4	21.5 ± 1.0
50430.999	3/2	5.3	6.6 ± 0.4
51045.292	7/2	4.3	4.7 ± 0.4
51254.429	3/2	2.2	2.1 ± 0.2
51438.064	5/2	4.7	4.6 ± 0.4
54498.608	7/2	2.9	2.1 ± 0.2
55392.446	9/2	2.4	2.3 ± 0.2
W III			
57231.04	2	2.9	2.9 ± 0.3
61488.36	3	2.3	2.5 ± 0.3

~ 25%

~ 10%

Comparison between theoretical oscillator strengths and available experimental results
(W II : Kling et al., JOSRT **67**, 227, 2000)
(W III : Schultz-Johanning et al., Phys. Scr. **63**, 367, 2001)



The specific case of tungsten (W IV, W V, W VI)

Transition rates in W IV, W V and W VI

W IV : 105 experimentally known energy levels within the 5d³, 5d²s, 5d6s², 5d²p and 5d6s6p configurations
(Kramida & Shirai, At. Data Nucl. Data Tables **95**, 305, 2009)

W V : 59 experimentally known energy levels within the 5d², 5d6s, 5d6p, 6s6p, 5d5f and 5d7p configurations
(Kramida & Shirai, At. Data Nucl. Data Tables **95**, 305, 2009)

W VI : 14 experimentally known energy levels within the 5d, 6s, 6p, 6d, 7s, 5f, 5g and 6g configurations
(Kramida & Shirai, At. Data Nucl. Data Tables **95**, 305, 2009)

Transition probabilities and oscillator strengths calculated for :

W IV lines in the range 143 – 990 nm (Enzonga Yoca et al., J. Phys. B **45**, 035001, 2012)

W V lines in the range 83 – 1494 nm (Enzonga Yoca et al., J. Phys. B **45**, 065001, 2012)

W VI lines in the range 38 – 1148 nm (Enzonga Yoca et al., J. Phys. B **45**, 035002, 2012)

Conclusions

New radiative data (lifetimes, transition probabilities, oscillator strengths) obtained for a large number of lines belonging to heavy neutral and lowly ionized atoms (fifth row, sixth row, lanthanides)
[~ 90 ions considered, ~ 700 lifetimes measured, ~ 100000 A-values calculated]

Semi-empirical model based on the relativistic Hartree-Fock method including core-polarization effects

Good agreement between theoretical approach and experimental results

New data very useful in astrophysics, plasma physics, ...

Level structure still too poorly known for many ions to perform semi-empirical calculations (→ new term analyses needed)

Collaborations

Atomic structure calculations

Astrophysics & Spectroscopy, UMONS, Belgium (* also ULg, Liège, Belgium)

(E. Biémont*, V. Fivet, P. Palmeri, P. Quinet*)

Department of Physics, University of Brazzaville, Congo

(S. Enzonga Yoca)

Experimental measurements

Department of Physics, Lund University, Lund, Sweden

(L. Engström, H. Hartman, Z. Li, H. Lundberg, H. Nilsson, S. Svanberg, H. Xu, Z. Zhang)

Department of Physics, Jilin University, Changchun, China

(Z. Dai, L. Han, Z. Jiang, P. Li, Z. Ma, G. Sun, S. You, J. Xu, W. Zhang, Y. Zhang)

Also collaboration with

Institute of Solid State Physics, Bulgarian Academy of Sciences, Sofia, Bulgaria

(K. Blagoev, G. Malcheva)

Faculty of Physics, Universidad Complutense de Madrid, Spain

(R. Mayo, M. Ortiz)

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Thank you
for your
attention !

